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Type IIA moduli stabilization

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Abstract: We demonstrate that flux compactifications of type IIA string theory can classically stabilize all geometric moduli. For a particular orientifold background, we explicitly construct an infinite family of supersymmetric vacua with all moduli stabilized at arbitrarily large volume, weak coupling, and small negative cosmological constant. We obtain these solutions from both ten-dimensional and four-dimensional perspectives. For more general backgrounds, we study the equations for supersymmetric vacua coming from the effective superpotential and show that all geometric moduli can be stabilized by fluxes. We comment on the resulting picture of statistics on the landscape of vacua.

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Contents

1. Introduction

It is an important problem to understand the effects which stabilize moduli in quasi-realistic string compactifications. Stabilized compactifications likely provide the correct setting both for stringy models of early universe cosmology, and for string-based models of particle phenomenology. In addition, the properties of the set of such vacua (perhaps endowed with a preferred cosmological measure) may suggest new predictions, or at least possible interesting phenomenological signatures, of string theory.

This problem has received a great deal of attention recently. In the framework of low-energy supersymmetry, the most concrete constructions have appeared in the IIB theory [1–3], while constructions which break supersymmetry at a high scale have been described in both critical and noncritical string theories [4]. Proposals for constructing stabilized models in the 11D, heterotic and type I limits have also appeared [5–8]. The range of constructions seems to be quite large, realizing the idea of a discretuum [9] and probably requiring statistical analysis to get a reasonable picture of the set of possibilities [10–12].

While the evidence for the existence of many stabilized vacua is quite suggestive, it is fair to say that it has been hard to come by extremely controlled individual examples. The main problem is that, by definition, any concrete example cannot have tunable couplings left over, since the string coupling and radii have been fixed. In the IIB context, it has proven possible to obtain supersymmetric vacua with weak string couplings, and radii which grow as the logarithm of a tuning parameter [1]; completely explicit examples appear in [2]. This leads to control, but only through fine tuning by appropriate choices in a large space of flux vacua. For nonsupersymmetric IIB vacua, it has been argued that one can obtain "large extra dimensions" as well by looking at scaling regimes for moduli where loop and non-perturbative corrections to the potential conspire to make this possible [3].

In this paper, we show that it is possible to construct stabilized vacua with arbitrarily weak coupling g_s and large radius R in the setting of type IIA Calabi-Yau compactifications with flux. We do this by demonstrating the existence of infinite families of vacua where g_s and R have power law dependence on a flux which is unconstrained by tadpoles, and asymptote to weak coupling and large radius in the large flux limit. Our solutions can be seen both directly from classical 10D supergravity and from the effective 4D framework developed in [13] and extended here. We note that it was anticipated in the papers [14, 13] that generic fluxes should stabilize the geometric moduli of IIA Calabi-Yau models,[1] and in [15] it was shown that untwisted moduli could be stabilized by fluxes in a particular IIA orientifold. The main advance here is to provide an example with all moduli stabilized, to make the generic stabilization of moduli explicit, and to demonstrate the existence of vacua with very large radius and weak coupling, where it is clear that all approximations are controlled.

In section 2, we introduce the simple toroidal orientifold compactification of type IIA string theory that will be our example. In section 3, we analyze this orientifold model in the presence of fluxes using type IIA supergravity in 10 dimensions, and show that the moduli are all classically stabilized. In section 4 we present a general analysis of

[1] General discussions of IIA compactifications on spaces with various G-structures appear in [16, 17].

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IIA compactifications from the point of view of $\mathcal{N} = 1$ supergravity in 4D, extending the earlier work of Grimm and Louis [13]. We show using this formalism that the classical stabilization of geometric moduli is generically possible in IIA orientifold compactifications, and demonstrate the generic existence of families of vacua admitting parametric control over the volume and string coupling. In section 5 we apply the general 4D analysis to the model of section 2 and relate the 4D and 10D pictures in this case. Section 6 contains a discussion of the properties of the landscape of IIA vacua and compares to other ensembles. We conclude in section 7. In the appendix we provide an elementary derivation of the type IIA Chern-Simons terms in the presence of background fluxes which are needed for our analysis.

2. A simple model: $T^6/\mathbb{Z}_3^2$

In this section we describe a simple type IIA orientifold compactification which we will use as an example throughout this paper. The model is a $T^6/\mathbb{Z}_3$ orientifold, modded out by an additional freely acting $\mathbb{Z}_3$ symmetry [18 − 20], preserving $\mathcal{N} = 1$ supersymmetry in four dimensions. A discussion of the stabilization of untwisted moduli for a $T^6/(\mathbb{Z}_2 \times \mathbb{Z}_2)$ IIA orientifold appears in [15].

This compactification has a fairly small number of moduli and is easy to analyze explicitly. There are moduli corresponding to the sizes of the three 2-tori $T^6 = T^2 \times T^2 \times T^2$, a B-field modulus for each T^2, and finally the dilaton and a single axion arising from the 3-form C_3; there are no complex structure moduli. Furthermore, there are additional metric and B-field moduli associated with blow-ups of 9 singular orbifold points.

In section 3, we show that all these moduli are stabilized in type IIA supergravity when the zero- and three-form fluxes F_0 and H_3 canceling the tadpole from the orientifold fixed plane are combined with generic four-form fluxes F_4. We demonstrate this by directly calculating the potential for the zero modes. In section 5, we consider the $\mathcal{N} = 1$ four-dimensional effective supergravity description and show that depending on the signs of the fluxes, these stabilized vacua may be supersymmetric solutions extremizing the flux superpotential.

Let us describe the orientifold in more detail. We parameterize the torus T^6 by three complex coordinates $z_i = x_i + iy_i$, subject to the periodicity conditions

$$z_i \sim z_i + 1 \sim z_i + \alpha \,, \tag{2.1}$$

where $\alpha = e^{\pi i/3}$. This torus has a $\mathbb{Z}_3$ symmetry T under the action

$$T : (z_1, z_2, z_3) \to (\alpha^2 z_1, \alpha^2 z_2, \alpha^2 z_3) \,. \tag{2.2}$$

This transformation has 27 fixed points, and the resulting orbifold is a singular limit of a Calabi-Yau with Euler character $\chi = 72$.[2] This orbifold was constructed in [18] and its geometry was analyzed in detail in [19], where it was also pointed out that the resulting

[2]Note that while homotopically nontrivial curves in the original T^6 (such as a cycle wrapping once on x_i) are not all trivial after the $\mathbb{Z}_3$ quotient, such nontrivial cycles project to elements of the $\mathbb{Z}_3$ around the fixed points and are removed when these points are blown up to form a smooth Calabi-Yau, so that the resulting CY indeed has no π_1.

space has a further $\mathbb{Z}_3$ symmetry acting without fixed points according to

$$Q : (z_1, z_2, z_3) \to \left(\alpha^2 z_1 + \frac{1+\alpha}{3},\ \alpha^4 z_2 + \frac{1+\alpha}{3},\ z_3 + \frac{1+\alpha}{3}\right). \tag{2.3}$$

Modding out by this additional $\mathbb{Z}_3$ leads to a singular limit of a Calabi-Yau with $\chi = 24$ having 9 $\mathbb{Z}_3$ singularities. This compactification has $h^{2,1} = 0$ and $h^{1,1} = 12$, with 9 of the 12 Kähler moduli arising from blow-up modes of the 9 singularities.

Following [20], we can construct an orientifold of this $T^6/\mathbb{Z}_3^2$ orbifold, modding out by $\mathcal{O} = \Omega_p(-1)^{F_L}\sigma$ where Ω_p is worldsheet parity, $(-1)^{F_L}$ is left-moving fermion number and σ is the reflection

$$\sigma :\ z_i \to -\bar{z}_i\,, \tag{2.4}$$

for each $i = 1, 2, 3$. This gives an $\mathcal{N} = 1$ supersymmetric type IIA orientifold model with an O6 orientifold plane filling the 4 noncompact directions and wrapping a 3-cycle on the T^6.

We are interested in the moduli of this orientifold compactification, corresponding to constant modes of the various supergravity fields that survive the orbifold and orientifold projections. Let us begin by discussing the metric on the T^6. Invariance of the metric under the action (2.3) of Q dictates that $g_{ij} = g_{\bar{i}\bar{j}} = g_{i\bar{j}} = 0$ if $i \neq j$ for $i, j = 1, 2, 3$. Further, from the invariance of the metric under the action (2.2) of T, it follows that the metric on each T^2 is diagonal. Thus, we can parameterize the metric on the compact space as

$$ds^2 = \sum_{i=1}^{3} \gamma_i\, dz^i d\bar{z}^i = \sum_{i=1}^{3} \gamma_i \left((dx^i)^2 + (dy^i)^2\right), \tag{2.5}$$

with 3 real moduli γ_i corresponding to the size of each of 3 T^2s; all other metric degrees of freedom, including all complex structure moduli, are projected out.

Consider now the invariant moduli associated with p-forms on the compact space. We can determine how the various modes of each p-form field transform under the two $\mathbb{Z}_3$ symmetries by using

$$T : dz_i \to \alpha^2 dz_i\,, \qquad Q : dz_i \to \alpha^{2i} dz_i\,. \tag{2.6}$$

The only two-forms which are invariant under both T and Q are $dz^i \wedge d\bar{z}^i$, which we use to construct a basis $\{w_i\}$, odd under the reflection σ (2.4),

$$w_i \;=\; (\kappa\sqrt{3})^{1/3} i dz^i \wedge d\bar{z}^i\,, \qquad \int_{T^6/\mathbb{Z}_3^2} w_1 \wedge w_2 \wedge w_3 \;\equiv\; \kappa\,, \tag{2.7}$$

where we have left the overall normalization of the triple intersection κ arbitrary. For later convenience we define a dual basis of even four-cycles $\{\tilde{w}^i\}$,

$$\tilde{w}^i \;=\; \left(\frac{3}{\kappa}\right)^{1/3} (idz^j \wedge d\bar{z}^j) \wedge (idz^k \wedge d\bar{z}^k)\,, \qquad \int_{T^6/\mathbb{Z}_3^2} w_i \wedge \tilde{w}^j \;=\; \delta_i^j\,, \tag{2.8}$$

where j and k are the two values of $1, 2, 3$ besides i.

The NSNS 2-form potential B_2 is odd under the world-sheet orientifold transformation $\Omega_p(-1)^{F_L}$; hence one may have nonzero

$$B_2 = \sum_{i=1}^{3} b_i\, w^i\,. \tag{2.9}$$

These real b_i combine with multiples of the γ_i,

$$v_i \;\equiv\; \frac{1}{2}\,\frac{1}{(\kappa\,\sqrt{3})^{1/3}}\;\gamma_i\,. \tag{2.10}$$

into three complex parameters which will be identified with the Kähler moduli of the four-dimensional supergravity studied in sections 4 and 5.

Because H^1 of the resolved orientifold is trivial, there are no moduli associated with the R-R one-form C_1. There is a single modulus associated with the dilaton ϕ, as well as its partner, an axion field ξ coming from the RR potential C_3, as we now describe.

The three-forms which are invariant under T and Q are the holomorphic 3-form

$$\Omega \;=\; 3^{1/4}\, i\, dz_1\wedge dz_2\wedge dz_3\,, \tag{2.11}$$

and its complex conjugate $\bar{\Omega}$. The normalization is fixed to satisfy the following convenient condition

$$i\int_{T^6/\mathbb{Z}_3^2} \Omega\wedge\bar{\Omega} \;=\; 1 \tag{2.12}$$

where we used $i\int_{T^2} dz_i\wedge d\bar{z}_i \;=\; \sqrt{3}$.

We can decompose Ω into real and imaginary components

$$\Omega \;=\; \frac{1}{\sqrt{2}}\,(\alpha_0 + i\,\beta_0)\,, \tag{2.13}$$

where because under (2.4) we have $\sigma:\Omega\to\bar{\Omega}$, α_0 and β_0 are even and odd respectively under orientifold reflection; the orientifold is hence wrapped on the α_0 cycle. They form a symplectic basis,

$$\int_{T^6/\mathbb{Z}_3^2} \alpha_0\wedge\beta_0 \;=\; 1\,. \tag{2.14}$$

Under the world-sheet orientifold transformation $\Omega_p(-1)^{F_L}$, $C_{(3)}$ is even, and hence the single modulus ξ of the R-R three-form is

$$C_{(3)} \;=\; \xi\,\alpha_0\,. \tag{2.15}$$

The axion ξ and the dilaton ϕ combine into the complex axiodilaton modulus.

In addition to the 4 complex moduli we have already described, 9 further (complex) Kähler moduli are associated with the blow-ups of the 9 singular points of the orientifold. Locally, each blow-up looks like a resolution of $\mathbb{C}^3/\mathbb{Z}_3$, and is parameterized by a scale modulus and a corresponding B-field modulus. Globally, these moduli can be described in terms of the metric and B-field degrees of freedom on a smooth Calabi-Yau whose singular limit is the $T^6/\mathbb{Z}_3$ orientifold.

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Although we do not have an explicit form for the metric on the smooth Calabi-Yau, we can give a local analysis of these blow-up modes from the point of view of 10D supergravity, which we do in section 3. Furthermore, in the 4-dimensional picture, the prepotential for these modes is known to leading order, allowing us to find solutions with all blow-up moduli stabilized; this analysis is carried out in Section 5.

3. Moduli stabilization of $T^6/\mathbb{Z}_3^2$ in classical IIA supergravity

We will now directly calculate from the massive type IIA supergravity action the potential for the moduli of the orientifold compactification presented in the previous section. In subsection 3.1 we describe the supergravity action on the orientifold in the presence of fluxes. Subsection 3.2 solves the equation of motion for the R-R seven-form field $C_{(7)}$ which fixes the tadpole cancellation condition. In subsection 3.3 we stabilize the bulk moduli of the compactification by solving the remaining supergravity equations of motion. Subsection 3.4 examines potential tachyonic directions, showing that although for some signs of fluxes there are tachyons, their masses do not exceed the Breitenlohner-Freedman bound, so that they do not represent true instabilities; the analysis of section 5 will show that only the vacua associated to certain choices of fluxes, all of which have no tachyons, are supersymmetric. Finally, subsection 3.5 contains a description of the stabilization of the blow-up modes.

3.1 Fluxes and the IIA supergravity action

In order to stabilize all moduli, we will turn on background fluxes on the orientifold. In addition, the orientifold produces a tadpole for the C_7 potential, which must be canceled either by wrapped D6-branes or fluxes; in the next section we will show how to satisfy the tadpole constraints with fluxes alone.

We will turn on a constant F_0, as well as NS-NS three-form flux H_3 and R-R four-form flux F_4. The first two are necessary to cancel the tadpole, and then the last completes the flux stabilization. For simplicity we leave $F_2 = 0$; we discuss the generalization to nonzero F_2 in sections 4 and 5, and find that most choices of F_2 are physically redundant under gauge transformations, while the few physically inequivalent vacua with nonzero F_2 have qualitatively identical behavior to the $F_2 = 0$ case we consider here. F_6 only comes into stabilizing the axion ξ.

Since B_2 is odd under the orientifold action, the three-form background H_3^{bg} must multiply the unique odd 3-form (2.13),

$$H_3^{\text{bg}} = -p\,\beta_0\,, \tag{3.1}$$

while the four-form flux F_4 is expanded in the basis (2.8) of even 4-cycles,

$$F_4^{\text{bg}} = e_i\,\tilde{w}^i\,. \tag{3.2}$$

We can also turn on four-form flux through 4-cycles associated with the blow-up modes, as we discuss in subsection 3.5.

The presence of nonzero F_0 means that instead of ordinary type IIA supergravity, we must use the massive type IIA theory [21], with mass parameter $m_0 = F_0$. The string frame action is then

$$S = S_{\rm kinetic} + S_{\rm CS} + S_{O6}\,, \tag{3.3}$$

where the action is decomposed into a Chern-Simons piece $S_{\rm CS}$, a piece from the orientifold $S_{\rm O6}$, and a "kinetic" piece (everything else). The kinetic terms are[3]

$$S_{\rm kinetic} = \frac{1}{2\kappa_{10}^2}\int d^{10}x\,\sqrt{-g}\left(e^{-2\phi}\left(R+4(\partial_\mu\phi)^2-\frac{1}{2}|H_3^{\rm total}|^2\right)-(|\tilde F_2|^2+|\tilde F_4|^2+m_0^2)\right), \tag{3.4}$$

where $2\kappa_{10}^2 = (2\pi)^7\alpha'^4$, with field strengths given by

$$\begin{aligned} H_3^{\rm total} &= dB_2 + H_3^{\rm bg}\,,\\ \tilde F_2 &= dC_1 + m_0 B_2\,,\\ \tilde F_4 &= dC_3 + F_4^{\rm bg} - C_1\wedge H_3 - \frac{m_0}{2}B_2\wedge B_2\,, \end{aligned} \tag{3.5}$$

and $|F_p|^2 = F_{\mu_1\ldots\mu_p}F^{\mu_1\ldots\mu_p}/p!$. We denote by B_2, C_3 only the fluctuation part of the form field around the given background flux. The Chern-Simons piece takes the form

$$\begin{aligned} S_{\rm CS} = -\frac{1}{2\kappa_{10}^2}\int\Big[& B_2\wedge dC_3\wedge dC_3 + 2B_2\wedge dC_3\wedge F_4^{\rm bg} + C_3\wedge H_3^{\rm bg}\wedge dC_3 - \\ & -\frac{m_0}{3}B_2\wedge B_2\wedge B_2\wedge dC_3 + \frac{m_0^2}{20}B_2\wedge B_2\wedge B_2\wedge B_2\wedge B_2\Big]\,. \end{aligned} \tag{3.6}$$

The separation of the usual $\int B_2\wedge F_4\wedge F_4$ Chern-Simons term into several pieces is needed because topological fluxes must appear in the field strengths, and in the presence of fluxes the second and third terms in (3.6) are not related by the usual integration by parts. An elementary derivation of the relevant terms from M-theory is given in the appendix. In principle there should be similar contributions involving the background fluxes in the massive IIA theory of the form $m_0 B^3 F_4^{\rm bg}$ and $m_0 B^2 H_3^{\rm bg} C_3$; we do not need such terms for the analysis here. Quantum type IIA string theory involves a number of subtleties related to the K-theoretic classification of branes and fluxes [23], some of which generalize the Chern-Simons terms [24]; these subtleties do not affect our results.

Finally, the contribution of the orientifold fixed plane to the action is given by

$$S_{\rm O6} = 2\mu_6\int_{O6} d^7\xi\, e^{-\phi}\sqrt{-g} - 2\sqrt{2}\mu_6\int C_{(7)}\,, \tag{3.7}$$

where $\mu_p = (2\pi)^{-p}\alpha'^{-(p+1)/2}$ is the Dp-brane charge and tension, and we have taken into account that the charge of an Op-plane is -2^{p-5} that of a Dp-brane.

Before proceeding to evaluate the C_7 tadpole, we remark on the quantization of the fluxes. For a canonically normalized F_p field strength, the usual (cohomological) quantization condition is

$$\int F_p = 2\kappa_{10}^2\,\mu_{8-p}\,f_p = (2\pi)^{p-1}\alpha'^{(p-1)/2}f_p\,, \tag{3.8}$$

[3]We follow the conventions of [13] for the RR fields (including m_0) so we can more easily match the 4D superpotential analysis; they are related to those of Polchinski [22] by $C_{RR} = C_{RR}^{Polch}/\sqrt{2}$.

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with f_p an integer; in our convention the expression for RR fields must be rescaled by a factor of $\sqrt{2}$. Hence we can write the fluxes we are using in terms of integers f_0, h_3, f_4^i as

$$m_0 = \frac{f_0}{2\sqrt{2}\pi\sqrt{\alpha'}}\,, \qquad p = (2\pi)^2\alpha'\, h_3\,, \qquad e_i = \frac{\kappa^{1/3}}{\sqrt{2}}\,(2\pi\sqrt{\alpha'})^3\, f_4^i\,. \tag{3.9}$$

The K-theoretic classification of fluxes [23] modifies the condition (3.8) in certain circumstances. The effect potentially relevant to our analysis is that when the first Pontryagin class divided by two $p_1/2$ of the tangent bundle of the compactification manifold is odd, the f_4^i are half-integers instead of integers [25]; however this shift turns out not to affect any of the cycles in our $T^6/\mathbb{Z}_3^3$ example, as p_1 is always divisible by four for a Calabi-Yau threefold.[4]

3.2 Cancelling the tadpole

As is evident from (3.7), the O6 plane generates a tadpole for the C_7-potential Hodge dual to C_1. This can be cancelled by adding 2 D6-branes for each O6, but instead we cancel it using the background fluxes.

One may show by analyzing the RR equations of motion and Bianchi identities, as well as various gauge invariances in the brane actions, that

$$\tilde{F}_6 \equiv *\tilde{F}_4 = dC_5 - C_3 \wedge H_3 + \frac{m_0}{6} B_2 \wedge B_2 \wedge B_2\,, \tag{3.10}$$

$$\tilde{F}_8 \equiv *\tilde{F}_2 = dC_7 - C_5 \wedge H_3 - \frac{m_0}{24} B_2 \wedge B_2 \wedge B_2 \wedge B_2\,. \tag{3.11}$$

The equation of motion for C_7 then receives contributions from the $|\tilde{F}_2|^2$ term in (3.4), as well as from the O6-plane in (3.7). Integrating over the β_0 cycle, one finds

$$\int d\tilde{F}_2 = 2\sqrt{2}\kappa_{10}^2\,\mu_6\,, \tag{3.12}$$

which using $d\tilde{F}_2 = m_0 H_3$ gives the tadpole condition

$$m_0 p = -2\sqrt{2}\kappa_{10}^2\,\mu_6 = -2(\sqrt{2}\pi\sqrt{\alpha'})\,. \tag{3.13}$$

Hence we learn that m_0 and p must be of opposite sign. We note that the quantization condition (3.9) requires

$$m_0 p = (\sqrt{2}\pi\sqrt{\alpha'}) f_0 h_3 \tag{3.14}$$

with f_0, h_3 integers, so that the minimal charge that can be obtained from $F_0 H_3$ is just that of a single D6-brane. To satisfy the tadpole (3.13) we have a very limited set of possibilities for the fluxes: $f_0 h_3 = -2 \to \pm(f_0, h_3) = (-1, 2), (-2, 1)$. Hence there is very little freedom to tune in the H_3, F_0 sector; what freedom we have will come from F_4.

We note that in other models with more 3-cycles, there will be in general $h^{2,1} + 1$ tadpole conditions to satisfy; this is to be contrasted with the single C_4 tadpole familiar in IIB flux compactifications. Thus once F_0 is nonzero, every mode of H_3 will be constrained by a tadpole condition.

[4] We thank Paul Aspinwall for comments on characteristic classes for Calabi-Yaus.

3.3 Stabilizing bulk moduli

Having chosen the H_3 and F_0 fluxes so as to satisfy the tadpole cancellation condition (3.13), we now turn to evaluating the potential for the moduli and solving the resulting equations of motion. We insert the background fluxes (3.1), (3.2) into the supergravity action, and write the metric, B field and 3-form field C_3 in terms of the bulk moduli using (2.5), (2.9), (2.15); to determine the potential we assume the modes γ^i, b^i, ϕ, ξ are coordinate-independent.

A more complete analysis would include the warp factors in the metric and the full dependence of the supergravity fields on the compact directions. We leave the details of such an analysis for future work; as we shall see, the model we are considering here admits solutions in a regime of large volume and weak coupling where these effects are unimportant.

We begin by considering the RR 3-form field C_3. The single modulus ξ of this field appears only in the Chern-Simons term $C_3 \wedge H_3^{bg} \wedge dC_3$, and we note that given the value (3.1) for H_3^{bg}, this term is only nonzero if the remaining dC_3 is polarized along the spacetime directions; hence treating this latter mode is necessary for determining the equation for the axion ξ. This field has no physical degrees of freedom; we shall call it $dC_3|_{4D} \equiv \mathcal{F}_0$ and treat it as a Lagrange multiplier. A more careful, quantum-mechanical treatment leading to the same result is described in [26].

For a field with couplings of the form

$$S = -\frac{1}{2\kappa_{10}^2} \int \left(\mathcal{F}_0 \wedge * \mathcal{F}_0 + 2\mathcal{F}_0 \wedge X \right) , \tag{3.15}$$

the equation of motion for $\mathcal{F}_0$ merely sets $\mathcal{F}_0 = *X$; substituting this back into the action, (3.15) becomes

$$S = -\frac{1}{2\kappa_{10}^2} \int X \wedge * X \, . \tag{3.16}$$

Hence minimization of these terms in the potential simply sets $X = 0$. Calculating X for the case at hand and integrating over the compact space, we have

$$\int X = 0 = \int \left(F_6^{\rm bg} + B_2 \wedge F_4^{\rm bg} + C_3 \wedge H_3^{\rm bg} - \frac{m_0}{6} B_2 \wedge B_2 \wedge B_2 \right) , \tag{3.17}$$

which evaluates to an equation for the 3-form axion ξ,

$$p\,\xi = e_0 + e_i b_i - \kappa\, m_0\, b_1 b_2 b_3 \, , \tag{3.18}$$

where we put $e_0 = \int F_6^{\rm bg}$.

We now solve the equation of motion for the B field components. Since there are no zero modes of C_1 and we have taken $F_2^{\rm bg} = 0$, the $|\tilde{F}_2|^2$ and $|\tilde{F}_4|^2$ terms are at least quadratic in b_i; the Chern-Simons terms have already been accounted for in the minimization of X. Thus (3.3) is at least quadratic in b^i, meaning we can consistently find a solution with $b_i = 0$.

Notice that the term $|\tilde{F}_4|^2$ gives rise to an off-diagonal quadratic term for the B-field moduli of the form $(F_4^{\rm bg})_{abcd}\, B^{ab} B^{cd}$. Such a term can lead to an unstable B mode. After solving for the rest of the moduli we return to this term in subsection 3.4 and check to see when the quadratic form for the B moduli is positive definite around the solution.

The moduli that remain are the sizes γ_i of the 2-tori and the dilaton ϕ; we now write the four-dimensional effective potential for these. We note first that to properly normalize the four-dimensional Einstein term, we pass to a 4D Einstein frame with the redefinition

$$g_{\mu\nu} = \frac{e^{2\phi}}{\text{vol}}\, g^E_{\mu\nu}\,, \tag{3.19}$$

for the four-dimensional metric only. We then define the effective potential V,

$$S = \frac{1}{\kappa_{10}^2}\int d^4x\sqrt{-g_E}(-V)\,, \tag{3.20}$$

and find the result

$$V \;=\; \frac{p^2}{4}\,\frac{e^{2\phi}}{\text{vol}^2} + \frac{1}{2}\left(\sum_{i=1}^{3} e_i^2\, v_i^2\right)\frac{e^{4\phi}}{\text{vol}^3} + \frac{m_0^2}{2}\,\frac{e^{4\phi}}{\text{vol}} - \sqrt{2}\,|m_0\, p|\,\frac{e^{3\phi}}{\text{vol}^{3/2}}\,, \tag{3.21}$$

where the four terms are from the $|H_3|^2$, $|\tilde{F}_4|^2$ and m_0^2 terms in (3.4) and the O6 Born-Infeld piece in (3.7), respectively; the $|\tilde{F}_2|^2$ and O6 Chern-Simons terms cancel according to the tadpole cancelation condition (3.13). We have defined the volume of compactification

$$\text{vol} \;\equiv\; \int_{T^6/\mathbb{Z}_3^2}\sqrt{g_6} \;=\; \frac{1}{8\sqrt{3}}\,\gamma_1\gamma_2\gamma_3 \;\equiv\; \kappa\, v_1 v_2 v_3\,, \tag{3.22}$$

and written (3.21) in terms of the rescaled metric components v_i (2.10). The evaluation of the O6-plane contribution to the potential (3.21),

$$V_{O6} = -2\kappa_{10}^2\,\mu_6\,\frac{e^{3\phi}}{\text{vol}^2}\int d^3x\sqrt{g_3}\,, \tag{3.23}$$

was carried out using the calibration formula [27] for special lagrangian 3-cycles, which for us reads

$$\int d^3x\sqrt{g_3} = 2\sqrt{2}\ \text{vol}^{1/2}\int \text{Re}\,\Omega = 2\ \text{vol}^{1/2}\int \alpha_0\,. \tag{3.24}$$

We now want to solve the equations

$$\frac{\partial V}{\partial\phi} = \frac{\partial V}{\partial v_i} = 0\,. \tag{3.25}$$

The structure of the ∂_{v_i} equations is

$$\frac{F(\text{vol},\phi)}{v_i} + e_i^2\, v_i\, G(\text{vol},\phi) = 0\,, \tag{3.26}$$

where F, G are some functions of vol and ϕ. Thus, we can reduce to two degrees of freedom using $v_i = v/|e_i|$, giving the simplified potential

$$V(D,v) \;=\; \frac{m_0^2}{2E}\,e^{4D}\,v^3 - \sqrt{2}\,|m_0\,p|\,e^{3D} + \frac{p^2}{4}\,\frac{e^{2D}}{v^3}\,E + \frac{3}{2}\,\frac{e^{4D}}{v}\,E\,, \tag{3.27}$$

where $E = |e_1 e_2 e_3|/\kappa$ ($\text{vol} = v^3/E$) and we have also introduced the 4-dimensional dilaton

$$e^D \;=\; \frac{e^\phi}{\text{vol}^{1/2}}\,. \tag{3.28}$$

Rescaling $e^D = |p|\sqrt{|m_0|/E}\,g$ and $v = \sqrt{E/|m_0|}\,r^2$, the potential becomes

$$\frac{1}{\lambda}V(g,r) = \frac{1}{2}g^4 r^6 - \sqrt{2}\,g^3 + \frac{1}{4}\frac{g^2}{r^6} + \frac{3}{2}\frac{g^4}{r^2} \tag{3.29}$$

where $\lambda = p^4 |m_0|^{5/2} E^{-3/2}$.

Now, we proceed to find the extremum of (3.29). We have

$$g\partial_g V + 2r\partial_r V = \lambda g^4 r^6 \left[4 - \frac{3}{\sqrt{2}}\left(\frac{1}{gr^6}\right) - \frac{5}{4}\left(\frac{1}{gr^6}\right)^2\right] = 0\,, \tag{3.30}$$

which implies $gr^6 = 5/(4\sqrt{2})$. Plugging in $g = 5/(4\sqrt{2}r^6)$ into $\partial_g V = 0$ gives $r^8 = 25/9$. We thus have the solution

$$\begin{aligned} v_i &= \frac{v}{|e_i|} = \frac{1}{|e_i|}\sqrt{\frac{5}{3}\left|\frac{e_1e_2e_3}{\kappa\, m_0}\right|}\,, \\ e^D &= |p|\sqrt{\frac{27}{160}\left|\frac{\kappa\, m_0}{e_1e_2e_3}\right|}\,, \end{aligned} \tag{3.31}$$

or equivalently in terms of the 10D metric and dilaton,

$$ds^2 = \left(\frac{1}{9\kappa}\right)^{1/6}\sqrt{5\left|\frac{e_1e_2e_3}{m_0}\right|}\,\sum_{i=1}^{3}\frac{1}{|e_i|}\,dz^i d\bar{z}^i\,, \tag{3.32}$$

$$e^\phi = \frac{3}{4}\,|p|\left(\frac{5}{12}\frac{\kappa}{|m_0e_1e_2e_3|}\right)^{1/4}. \tag{3.33}$$

Note that the κ dependence cancels out when the e_i are expressed in terms of the quantized fluxes (3.9).

One can show that

$$6g\partial_g V - r\partial_r V = 18V + 12\lambda\,\frac{g^4}{r^2}\,. \tag{3.34}$$

Thus, for the solutions (3.31) satisfying $\partial_g V = \partial_r V = 0$, the energy V is always negative:

$$V = -\frac{2E}{3v}\,e^{4D}\,, \tag{3.35}$$

and the 4D space-time is anti-de Sitter.

The solutions (3.31) stabilize all moduli for any choice of m_0, p satisfying the tadpole condition (3.13) and any four-form fluxes e_i. Because the four-form flux parameters e_i are not constrained by the tadpole, we have an infinite family of IIA vacua with this orientifold compactification.

The shape of the potential and AdS minima are exhibited in figures 1 and 2. Note that the finite distance minima for dimensionless variables r and g will correspond to minima at parametrically large radius and small coupling in terms of dimensionful parameters.

Scaling all the e_i as $e_i \sim \bar{e}$, we find that the metric components γ_i scale as $\bar{e}^{1/2}$ and hence the volume goes as $\bar{e}^{3/2}$, while the string coupling $e^\phi \sim \bar{e}^{-3/4}$ and the vacuum energy goes as $-\bar{e}^{-9/2}$. Thus, the infinite family of compactifications has parametrically increasing

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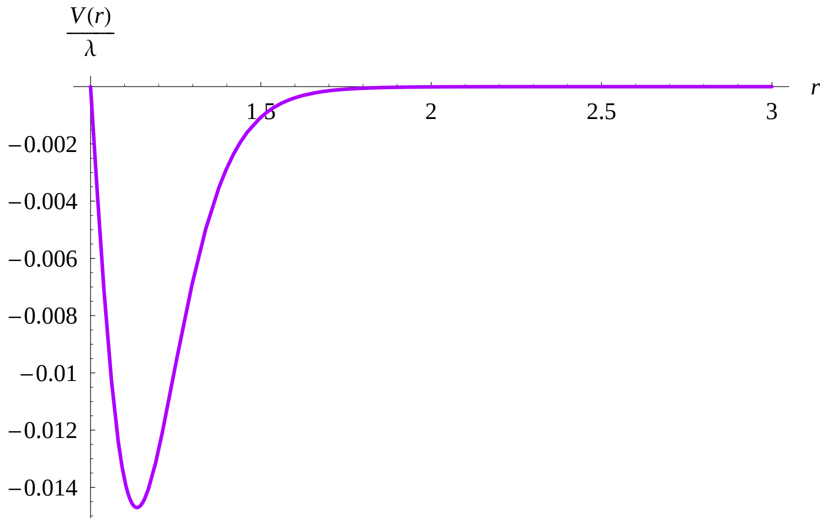

Figure 1: The potential $\frac{1}{\lambda}V(r,g)$ on solutions for g as a function of r.

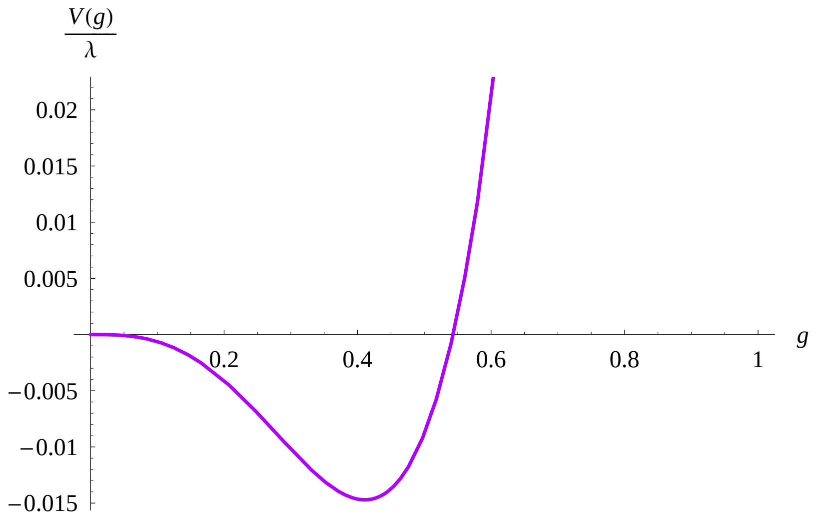

Figure 2: The potential $\frac{1}{\lambda}V(r,g)$ on solutions for r as a function of g.

volume and decreasing string coupling. As we will discuss further in section 5.2, the solutions are effectively four-dimensional at low energies, unlike the familiar Freund-Rubin models, which also arise in infinite families. This is (granting the controlled stabilization of the blow-up modes, which we discuss in section 3.5) the primary result of this paper: a class of four-dimensional vacua with all moduli stabilized by fluxes in a controlled regime where corrections can be made arbitrarily small.

3.4 Stability analysis

Because the e_i appear quadratically in the potential (3.21), the solution (3.31) exists for any choice of sign on the four-form fluxes; this is manifested by the absolute values in the solution. The sign of m_0 is also arbitrary, although from (3.13) we must have $\mathrm{sgn}(m_0 p) < 0$.

As we shall see in section 5, not all choices of sign for the fluxes lead to supersymmetric vacua at large volume. This suggests that some of the solutions (3.31) could have instabilities. We now consider the quadratic form for the fields B_2 and C_3 around the solutions (3.31) and look for possible tachyonic modes.

The B_2 field appears in the $|\hat{F}_2|^2$ and $|\tilde{F}_4|^2$ terms of (3.4). In the background given by the solution (3.31), these terms give contributions quadratic in B_2 of the form

$$-\frac{1}{2\kappa_{10}^2}\int d^{10}x\sqrt{-g}\,(|\hat{F}_2|^2+|\tilde{F}_4^2|) \to -\frac{1}{2\kappa_{10}^2}\int\left(m_0^2\,B_2\wedge *B_2 - m_0\,B_2\wedge B_2\wedge *F_4^{\mathrm{bg}}\right), \tag{3.36}$$

while from eliminating the Lagrange multiplier $\mathcal{F}_0$, we derive a mixing of B_2 with C_3 fluctuations (3.16), (3.17). Hence we also need the kinetic terms for both,

$$\begin{aligned}&-\frac{1}{2\kappa_{10}^2}\int d^{10}x\sqrt{-g}\left(\frac{1}{2}e^{-2\phi}|H_3|^2+|\tilde{F}_4|^2\right)\to\\ &\qquad\to -\frac{1}{2\kappa_{10}^2}\int\left(\frac{1}{2}e^{-2\phi}\,dB_2\wedge *dB_2 + dC_3\wedge *dC_3\right).\end{aligned} \tag{3.37}$$

These expressions lead to the quadratic action for b_i and ξ fluctuations around the background (3.31),

$$\begin{aligned}S_{\text{axion}} = \frac{1}{2\kappa_{10}^2}\int d^4x\sqrt{-g_E}\Bigg(&\sum_{i=1}^{3}\left[-\frac{1}{2}\partial_\mu\tilde{b}_i\partial^\mu\tilde{b}_i - e^{4D}\left(m_0^2\,\mathrm{vol}\,\tilde{b}_i^2 - 2\,m_0\,\tilde{b}_1\tilde{b}_2\tilde{b}_3\,\frac{e_i v_i}{\tilde{b}_i}\right)\right]-\\ &-\frac{1}{2}\partial_\mu x\partial^\mu x - \frac{e^{4D}}{\mathrm{vol}}\left(\tilde{b}_1 e_1 v_1+\tilde{b}_2 e_2 v_2+\tilde{b}_3 e_3 v_3-\frac{p}{\sqrt{2}}e^{-D}x\right)^2\Bigg),\end{aligned}$$

where we have normalized the kinetic terms by defining $\tilde{b}_i \equiv b_i/v_i$, $x \equiv \sqrt{2}e^D\xi$. The mass-squared matrix for the coupled $\tilde{b}_i$, x sector is then

$$M_{ij}^2 = 2\,|m_0|\,e^{4D}\,v\begin{pmatrix}\frac{34}{15} & \frac{3}{5}s_1s_2-s_3 & \frac{3}{5}s_1s_3-s_2 & \frac{4}{5}s_1\\ \frac{3}{5}s_1s_2-s_3 & \frac{34}{15} & \frac{3}{5}s_2s_3-s_1 & \frac{4}{5}s_2\\ \frac{3}{5}s_1s_3-s_2 & \frac{3}{5}s_2s_3-s_1 & \frac{34}{15} & \frac{4}{5}s_3\\ \frac{4}{5}s_1 & \frac{4}{5}s_2 & \frac{4}{5}s_3 & \frac{16}{15}\end{pmatrix}, \tag{3.38}$$

where $s_i \equiv \mathrm{sgn}(m_0e_i) = \pm 1$. It is easy to check that when

$$e_1e_2e_3m_0 < 0\,, \tag{3.39}$$

the matrix M_{ij}^2 is positive definite, whereas when $e_1e_2e_3m_0 > 0$, M_{ij}^2 has a negative eigenvalue, $M_{tachyon}^2 = -(2/15)(2\,|m_0|\,e^{4D}\,v)$.

Because the vacuum solutions are in anti-de Sitter space, it is not enough to find that a tachyonic mode exists for an instability to be present; tachyons whose negative mass-squared is above (less negative than) the Breitenlohner-Freedman bound [28],

$$m^2 \geq m_{BF}^2 \equiv -\frac{3}{4}|V|\,, \tag{3.40}$$

do not generate an unstable perturbation. Using (3.35), we find

$$\frac{M^2_{\text{tachyon}}}{m^2_{BF}} = \frac{8}{9}\,. \tag{3.41}$$

Since this is less than 1, the tachyon satifies the Breitenlohner-Freedman bound, and does not lead to an instability. Notice that this ratio is independent of the magnitudes of any of the fluxes; in general all the quadratic b and ξ fluctuations have their masses set by the AdS scale alone.

In principle, tachyons may also arise in the spectrum of metric or dilaton fluctuations. It is fairly straightforward to expand all metric and dilaton modes around the solution and confirm that the resulting mass matrix is positive definite.

Thus, all of our solutions (3.31) are perturbatively stable. We shall see in section 5 that only for certain signs of the fluxes are the solutions supersymmetric. The possible existence of quantum instabilities in these vacua is an interesting question we leave for the future.

3.5 Blow-up modes

Before closing this section, we discuss stabilizing the Kähler moduli associated to blow-up modes of the $T^6/\mathbb{Z}_3^2$ orbifold. A complete treatment of these degrees of freedom from ten-dimensional supergravity is difficult because we do not know the explicit form of the metric for the smooth Calabi-Yau which arises when all the singularities are blown up. In this section we simply consider the blow-up modes locally and show that they can be stabilized by a 4-form flux on the $\mathbb{CP}^2$ cycle which blows up the local $\mathbb{C}^3/\mathbb{Z}_3$ singularity.

The local analysis we carry out here is valid as long as the scale of the blow-up mode is much smaller than the scale of the compactification determined by the untwisted modes (while still very large in string units, so supergravity can be trusted). As we shall see, this can be guaranteed by choosing the flux on the $\mathbb{P}^2$ to be small compared to the untwisted fluxes e_i. Because we are already making a local approximation, we drop constant factors of order 1 in the analysis here and simply find the general form of the stabilized blow-up modes. In section 5 we consider the complete set of blow-up modes from the 4D supergravity picture, where all information needed to find the precise global form of the supergravity solution is contained in the prepotential.

Consider the noncompact $\mathbb{C}^3/\mathbb{Z}_3$ singularity. The resolution of this singularity by a $\mathbb{P}^2$ gives a one-parameter family of metrics on a line bundle $\mathcal{O}(-3)$ over $\mathbb{P}^2$. An explicit form of the metric is given by [29]

$$ds^2 \;=\; \frac{r^2}{2}\, g^{\text{FS}}_{i\bar{j}}\, dz_i d\bar{z}_j + F(r)^{-1}dr^2 + \frac{r^2}{9}\, F(r)(d\theta - 3A)^2\,, \tag{3.42}$$

where $F(r) = 1 - a^6/r^6$, a parameterizes the blow-up ($r \geq a$ for any fixed a), $g^{\text{FS}}_{i\bar{j}}$ is the Fubini-Study metric on $\mathbb{P}^2$, and A is a one-form with $dA = ig^{\text{FS}}_{i\bar{j}} dz_i \wedge d\bar{z}_j$. We want to put an integral flux f on the $\mathbb{P}^2$ and consider the effect on the 4D potential when this local blow-up occurs inside a much larger compact manifold.

The only terms in the 10D supergravity action (3.4) which are relevant are the m_0^2 and $|\tilde{F}_4|^2$ terms. As in the case of the bulk moduli, when $F_2 = 0$ we can consistently set $B_2 = 0$. There are potentially tachyons arising from new $F_4 B_2^2$ terms. There is only a single F_4 and a single B_2; the corresponding cubic intersection form on the blow-up cycle is nonzero (as we discuss in more detail in Section 5), so the condition that the vacuum be tachyon-free fixes the sign of the 4-form flux allowed. The m_0^2 term will, as in (3.21), take the form $m_0^2\, e^{4\phi}/\mathrm{vol}$ where vol is the total volume of the compactification.

From the form of the metric, we see that at blow-up parameter a we have roughly removed a region of radius a and volume a^6 from the volume vol_0 of the full compactification with no blow-up. More precisely, neglecting the cross-terms $d\theta \wedge A$, the volume form is $\sqrt{g} = r^6 \sqrt{g^{\mathrm{FS}}}/18$. Corrections to this volume form are small near $r \sim a$, where the major deformation away from the singular metric occurs. Thus, the correction to the volume is $\mathcal{O}(a^6)$ and so the volume is $\mathrm{vol} \sim \mathrm{vol}_0 - B\, a^6$ where B is a constant.

We can treat the $|\tilde{F}_4|^2$ term similarly. Because (neglecting backreaction) the four-form flux is on the $\mathbb{P}^2$, we have $|\tilde{F}_4|^2 \sim r^{-8} f^2$. Integrating this over the volume gives $\int_a r^{-3} \sim 1/a^2$, and using (3.19) we then have a total potential of the form

$$V_{\mathrm{blow-up}} \sim m_0^2 \frac{e^{4\phi}}{\mathrm{vol}} + C\, f^2 \frac{e^{4\phi}}{a^2\, \mathrm{vol}^2}\,, \tag{3.43}$$

where C is a constant and $\mathrm{vol} \sim \mathrm{vol}_0 - B\, a^6$. The minimum of the potential for a is then (for small a)

$$a^8 \sim \frac{C\, f^2}{B\, m_0^2}\,. \tag{3.44}$$

Thus, we see that

$$a \sim \left(\frac{f}{m_0}\right)^{1/4}. \tag{3.45}$$

We see that as long as $f \ll \bar{e}$ we have stabilized the blow-up mode at a scale much smaller than the untwisted moduli parameterizing the size of the overall compactification. So working in the regime $m_0 \ll f \ll \bar{e}$, we can accomplish controlled stabilization of the blow-up modes, in a regime where the supergravity approximation is valid. We derive the precise formula for the stabilized blow-up moduli in section 5 using the four-dimensional approach.

4. IIA flux vacua in 4D $\mathcal{N} = 1$ supergravity

The orientifold of $T^6/\mathbb{Z}_3^2$ we have studied so far is a particular case of the general class of $\mathcal{N} = 1$ supersymmetric orientifolds of Calabi-Yau compactifications of type IIA string theory. The effective theory of these models is an $\mathcal{N} = 1$ four-dimensional supergravity, characterized by a superpotential W generated by the fluxes for the moduli fields surviving the orientifold projection.

In this section, we review the derivation of the flux superpotential by Grimm and Louis [13] (for earlier related work see [30]; the form of these superpotentials was proposed in [31] and also derived in [15]), and then analyze the general structure of the supersym-

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metric vacua corresponding to solutions of the conditions $DW = 0$. The equations for the Kähler moduli decouple from the other fields and can be solved separately, as do the equations for the complex structure moduli; the dilaton is then fixed by an equation involving expectation values for the rest of the fields.

We show that in general, all geometric moduli can be frozen by fluxes; axionic partners of the complex structure moduli arising from C_3 remain unfixed, however. In the next section, we turn this analysis on the example of the $T^6/\mathbb{Z}_3^2$ orientifold, and find results in agreement with the previous sections.

4.1 Orientifold projection on $\mathcal{N} = 2$ moduli

The four-dimensional effective theory of type IIA string theory on a Calabi-Yau threefold is an $\mathcal{N} = 2$ supergravity. The moduli space is a product of two factors, one containing the vector multiplets (the Kähler moduli) and the other the hypermultiplets (the complex structure moduli and dilaton); the metric on each factor is determined by a Kähler potential. The orientifold projection to an $\mathcal{N} = 1$ theory reduces the size of each moduli space, as we review below.

The orientifold projection $\mathcal{O} = \Omega_p(-1)^{F_L}\sigma$ is the composition of worldsheet parity Ω_p, left-moving fermion number $(-1)^{F_L}$ and an antiholomorphic involution of the Calabi-Yau σ. The involution must act on the Kähler form J and holomorphic 3-form Ω as

$$\sigma^* J = -J\,, \qquad \sigma^*\Omega = e^{2i\theta}\overline{\Omega}\,, \tag{4.1}$$

where θ is some phase. The fixed loci of σ are special lagrangian three-cycles Σ_n satisfying

$$J|_{\Sigma_n} = 0\,, \qquad \mathrm{Im}(e^{-i\theta}\Omega)|_{\Sigma_n} = 0\,. \tag{4.2}$$

Orientifold six-planes (O6s) fill spacetime and wrap the Σ_n. One may always eliminate θ by a redefinition of Ω, and we shall do so in the following.

For modes of the massless ten-dimensional fields to be invariant under the orientifiold projection, they must transform under the antiholomorphic involution as

$$\sigma^* g_{\mu\nu} = g_{\mu\nu}\,, \qquad \sigma^* B_2 = -B_2\,, \quad \sigma^*\phi = \phi\,, \qquad \sigma^* C_1 = -C_1\,, \qquad \sigma^* C_3 = C_3\,. \tag{4.3}$$

4.1.1 Kähler moduli space

Before the orientifold projection, the vector multiplet moduli space is $h^{1,1}$-dimensional, the moduli corresponding to the expansion of the complexified Kähler form

$$J_c \equiv B_2 + iJ\,, \tag{4.4}$$

in a basis of $(1,1)$-forms. Under the projection, the space of $(1,1)$-forms $H^{1,1}$ decomposes into even and odd subspaces, $H^{1,1} = H^{1,1}_+ \oplus H^{1,1}_-$, of dimensions $h^{1,1}_-$ and $h^{1,1}_+ = h^{1,1} - h^{1,1}_-$, respectively. From (4.3) we see that the surviving modes of J_c are associated with odd forms, and hence we find $h^{1,1}_-$ surviving complex moduli t_a:

$$J_c = \sum_{a=1}^{h^{1,1}_-} t_a w_a\,, \qquad t_a = b_a + i v_a\,, \tag{4.5}$$

with $\{w_a\}$ a basis for $H^{1,1}_-$.

Hence the orientifold reduces the Kähler moduli space of the $\mathcal{N} = 2$ theory to a subspace without disturbing the moduli space complex structure. The Kähler potential for the reduced space is simply inherited from the $\mathcal{N} = 2$ theory:

$$K^K(t_a) = -\log\left(\frac{4}{3}\int J \wedge J \wedge J\right) = -\log\left(\frac{4}{3}\kappa_{abc}v_a v_b v_c\right), \tag{4.6}$$

where we defined the triple intersection

$$\kappa_{abc} \equiv \int w_a \wedge w_b \wedge w_c\,. \tag{4.7}$$

There are also $\mathcal{N} = 1$ vector multiplets associated to $H^{1,1}_+$ that survive the projection, but these contain no scalars and will not interest us.

4.1.2 Complex structure moduli space

Before the projection, the hypermultiplet moduli space is quaternionic. To define the complex structure moduli, as usual one chooses a basis for harmonic 3-forms H^3, $\{\alpha_{\hat{K}}, \beta_{\hat{L}}\}$, where $\hat{K}, \hat{L} = 0 \ldots h^{2,1}$ and $\int \alpha_{\hat{K}} \wedge \beta_{\hat{L}} = \delta_{\hat{K},\hat{L}}$. One can expand the holomorphic 3-form in this basis,

$$\Omega = Z_{\hat{K}}\alpha_{\hat{K}} - g_{\hat{L}}\beta_{\hat{L}}\,, \tag{4.8}$$

and the complex $Z_{\hat{K}}$ can be taken as homogeneous coordinates on the complex structure moduli space; we may call the inhomogeneous coordinates z_K, $K = 1 \ldots h^{2,1}$. The complex space of the z_K is promoted to a quaternionic space as each z_K is joined by the axionic modes ξ_K, $\tilde{\xi}_K$ defined as

$$C_3 = \xi_{\hat{K}}\alpha_{\hat{K}} - \tilde{\xi}_{\hat{L}}\beta_{\hat{L}}\,, \tag{4.9}$$

while ξ_0 and $\tilde{\xi}_0$ combine with the dilaton ϕ and the dual of B_2 polarized along spacetime to form the universal hypermultiplet. The moduli space is thus $4(h^{2,1} + 1)$-real dimensional.

Under the orientifold, the relevant space of harmonic forms again decomposes into even and odd subspaces, $H^3 = H^3_+ \oplus H^3_-$, where each of H^3_+ and H^3_- is $h^{2,1}+1$-real dimensional. The even and odd bases are $\{\alpha_k, \beta_\lambda\}$ and $\{\alpha_\lambda, \beta_k\}$, respectively, where $k = 0 \ldots \tilde{h}$ and $\lambda = \tilde{h} + 1 \ldots h^{2,1}$; the parameter $\tilde{h}$ determining how many αs are even is basis-dependent. The orientifold condition (4.1) with $\theta = 0$ requires

$$\mathrm{Im}\, Z_k = \mathrm{Re}\, g_k = \mathrm{Re}\, Z_\lambda = \mathrm{Im}\, g_\lambda = 0\,. \tag{4.10}$$

Two of these conditions are constraints on the moduli, while the other two follow automatically for a space admitting the antiholomorphic involution σ. We see that for each complex z_k, only one real component survives the projection. The condition (4.3) that C_3 be even also truncates the space of axion fields in half to $\{\xi_k, \tilde{\xi}_\lambda\}$. Consequently for each quaternionic modulus, one complex field survives: a real or imaginary part of the complex structure modulus and an axion.

The universal hypermultiplet is also cut in half, as ϕ and one of $\xi_0, \tilde{\xi}_0$ survive. One can summarize all the surviving moduli in the object

$$\Omega_c \equiv C_3 + 2i\,\mathrm{Re}(C\Omega)\,, \tag{4.11}$$

where the "compensator" C incorporates the dilaton dependence through

$$C \equiv e^{-D+K^{\rm cs}/2}\,, \qquad e^D \equiv \sqrt{8}e^{\phi+K^K/2} = \frac{e^\phi}{\sqrt{\rm vol}}\,. \tag{4.12}$$

Here e^D is the four-dimensional dilaton, equivalent to the previous definition (3.28) using $\int J\wedge J\wedge J = 6\,{\rm vol}$, and $K^{\rm cs}$ is the Kähler potential for complex structure moduli restricted to the surviving modes

$$K^{\rm cs} = -\log\left(i\int \Omega\wedge\overline{\Omega}\right) = -\log 2\left({\rm Im}\, Z_\lambda\, {\rm Re}\, g_\lambda - {\rm Re}\, Z_k\, {\rm Im}\, g_k\right). \tag{4.13}$$

The surviving moduli are then the expansion of Ω_c in H^3_+:

$$N_k \equiv \frac{1}{2}\int \Omega_c\wedge\beta_k = \frac{1}{2}\xi_k + i\,{\rm Re}(CZ_k)\,, \tag{4.14}$$

$$T_\lambda \equiv i\int \Omega_c\wedge\alpha_\lambda = i\tilde{\xi}_\lambda - 2\,{\rm Re}(Cg_\lambda)\,. \tag{4.15}$$

Note that including the dilaton via C means all $h^{2,1}+1$ complex modes are physical; the compensator essentially trades the irrelevant scale factor of Ω for the physically relevant dilaton field.

Thus in contrast to the Kähler case, where $h^{1,1}_-$ complex moduli are preserved and the rest removed, for the hypermultiplet moduli space each quaternion is cut in half, leaving always $h^{2,1}+1$ complex moduli. How many are N_k and how many are T_λ is basis-dependent; there is always a basis where $\tilde{h} = h^{2,1}$, and all moduli are N_k, leaving the real parts of the complex structure moduli, the ξ_k and the dilaton.

The Kähler potential for the surviving fields is

$$K^Q = -2\log\left(2\int {\rm Re}(C\Omega)\wedge * {\rm Re}(C\Omega)\right) = 4D\,, \tag{4.16}$$

where in the last step one used the identity

$$\int {\rm Re}(C\Omega)\wedge * {\rm Re}(C\Omega) = {\rm Im}(CZ_\lambda)\,{\rm Re}(Cg_\lambda) - {\rm Re}(CZ_k)\,{\rm Im}(Cg_k) = e^{-2D}/2\,, \tag{4.17}$$

derived using (4.13) and the definition (4.12) of D.

4.2 Fluxes and superpotential

One may turn on nonzero fluxes of the NSNS and RR field strengths consistent with the orientifold projection. Using (4.3), we find H_3 and F_2 must be odd, while F_4 is even. We write the fluxes as

$$H_3 = q_\lambda\alpha_\lambda - p_k\beta_k\,, \qquad F_2 = -m_a w_a\,, \qquad F_4 = e_a\tilde{w}^a\,, \qquad F_0 = m_0\,, \tag{4.18}$$

where we have used the fact that $H^{2,2}_+$ is the Poincaré dual of $H^{1,1}_-$ since the volume form $J\wedge J\wedge J$ is odd. The F_0 flux m_0 is the mass parameter of massive type IIA supergravity; an additional parameter $e_0 = \int F_6$ will arise as well.

Dimensionally reducing the massive IIA supergravity, neglecting the backreaction of the fluxes and other local sources, it was shown by Grimm and Louis in [13] that the resulting potential can be written in the form

$$V = e^K \left(\sum_{i,j,=\{t_a,N_k,T_\lambda\}} K^{ij} D_i W \overline{D_j W} - 3|W|^2 \right) + m_0 e^{K_Q} \operatorname{Im} W^Q \,, \tag{4.19}$$

where $K = K^K + K^Q$, and where the superpotential W is given by

$$W(t_a, N_k, T_\lambda) = W^Q(N_k, T_\lambda) + W^K(t_a) \,, \tag{4.20}$$

$$\begin{aligned} W^Q(N_k, T_\lambda) &= \int \Omega_c \wedge H_3 = -2p_k N_k - i q_\lambda T_\lambda \,, \\ &= -p_k \xi_k + q_\lambda \tilde{\xi}_\lambda + 2i \left[-p_k \operatorname{Re}(CZ_k) + q_\lambda \operatorname{Re}(Cg_\lambda) \right] \,, \end{aligned} \tag{4.21}$$

$$\begin{aligned} W^K(t_a) &= e_0 + \int J_c \wedge F_4 - \frac{1}{2} \int J_c \wedge J_c \wedge F_2 - \frac{m_0}{6} \int J_c \wedge J_c \wedge J_c \,, \\ &= e_0 + e_a t^a + \frac{1}{2} \kappa_{abc} m_a t_b t_c - \frac{m_0}{6} \kappa_{abc} t_a t_b t_c \,, \end{aligned} \tag{4.22}$$

with D_i the Kähler covariant derivative $D_i W \equiv \partial_i W + W \partial_i K$. The constant term e_0 comes from the space-time dual of F_4 polarized in the noncompact directions, as in Section 3.3 and as discussed in more detail in [26, 13], but may equivalently be thought of as the integrated flux of F_6.

When the tadpole conditions are satisfied, the last term in (4.19) cancels with contributions from local (O6 and D6) sources, and hence is absent in the total potential.[5] Consequently, the potential is completely characterized by the superpotential W (4.20).

4.3 Supersymmetric vacua

The superpotential (4.20) was derived by comparing to the dimensionally reduced ten-dimensional supergravity theory neglecting backreaction. For a general background, such an approximation cannot be used; not only will backreaction complicate the analysis, but contributions such as worldsheet instanton corrections that cannot be described in the language of ten-dimensional supergravity will appear. Corrections of this nature can be described naturally in the four-dimensional language; for example, worldsheet instanton corrections to the Kähler potential are well-known and can in some cases be calculated. In the regime of validity of effective field theory, the most useful description of the system is in terms of the four-dimensional quantities W, K, which one may use to attempt to determine the vacua and dynamics in terms of the properly corrected superpotential and Kähler potential.

Supersymmetric vacua are characterized by the vanishing of the F-term conditions,

$$D_{t_a} W = D_{N_k} W = D_{T_\lambda} W = 0 \,. \tag{4.23}$$

[5] This term and the local contributions against which it cancels were not mentioned in [13].

In this subsection we consider the general structure of these equations, and show that in general all geometric moduli can be fixed by fluxes. We shall focus on the regime of large volumes, where a geometric description is possible; however as described above, these equations can also be applied to the small volume region if the corrections are known.

4.3.1 Complex structure equations

The complex structure equations $D_{N_k}W = D_{T_\lambda}W = 0$ become

$$p_k + 2i\, e^{2D}\, W\ \mathrm{Im}(Cg_k) = 0\,, \tag{4.24}$$

$$q_\lambda + 2i\, e^{2D}\, W\ \mathrm{Im}(CZ_\lambda) = 0\,. \tag{4.25}$$

The first observation is that the imaginary part of each of these equations is identical. Given that C and D are real, one simply finds

$$\mathrm{Re}\, W = q_\lambda \tilde{\xi}_\lambda - p_k \xi_k + \mathrm{Re}\, W^K = 0\,. \tag{4.26}$$

This turns out to be the unique condition from (4.23) involving the axions. As a result, only a single linear combination of the ξ_k, $\tilde{\xi}_\lambda$ fields is fixed; the remaining ξ_k, $\tilde{\xi}_\lambda$ fields are the only moduli that cannot be stabilized using fluxes.

This collapse of what was apparently $h^{2,1} + 1$ constraints into a single constraint can be traced to the fact that the constant coefficients p_k, q_λ are real, and therefore do not contain enough degrees of freedom to stabilize both the complex structure moduli and the associated axions. A similar thing happens in the case of G_2 flux vacua [32]; we compare these ensembles in section 6.

We found the same result in section 3.3, where the only constraint on the axions arises from the space-time polarized F_4 through the Chern-Simons term $\int H_3 \wedge C_3 \wedge F_4$. This is not an issue in our $T^6/\mathbb{Z}_3^2$ example because there $h^{2,1} = 0$, so the single constraint suffices to fix the single axion arising from the dilaton multiplet. In more general examples, euclidean D2 instantons are expected to lift the remaining axions [33, 27]. In fact in this general class of models, the allowed H_3 fluxes live in the cohomology group H^3_- while the axions come from H^3_+. Hence the instantons which lift these axions are precisely the ones allowed by both the orientifold projection and by the nontrivial fluxes.

Turning to the real parts of (4.24), we note that $\mathrm{Im}\, W = 0$ is incompatible with any nonzero H_3 flux; we will see upon studying the Kähler sector that $\mathrm{Im}\, W \neq 0$ when any RR fluxes are turned on, as long as $\int J \wedge J \wedge J \neq 0$. Given nonzero $\mathrm{Im}\, W$, we find that if any p_k or q_λ vanishes, the corresponding modulus $\mathrm{Im}\, g_k$ or $\mathrm{Im}\, Z_k$ must vanish. Then for any k_i or λ_j with nonzero p_{k_i}, q_{λ_j}, we can eliminate $e^D\ \mathrm{Im}\, W$ to obtain

$$e^{-K^{\mathrm{cs}}/2}\, \frac{p_{k_1}}{\mathrm{Im}\, g_{k_1}} = e^{-K^{\mathrm{cs}}/2}\, \frac{p_{k_2}}{\mathrm{Im}\, g_{k_2}} = \ldots = e^{-K^{\mathrm{cs}}/2}\, \frac{q_{\lambda_1}}{\mathrm{Im}\, Z_{\lambda_1}} = \ldots \equiv Q_0\,. \tag{4.27}$$

These equations are invariant under an overall rescaling of Ω and hence depend only on the inhomogeneous coordinates on the complex structure moduli space; combined with the vanishing of $\mathrm{Im}\, g_k$ or $\mathrm{Im}\, Z_k$ for the cases when $p_k, q_\lambda = 0$, they constitute $h^{2,1}$ real equations that will in general fix all the $h^{2,1}$ complex structure moduli, independent of the RR fluxes or values of the Kähler moduli. The final equation from (4.24) can then be cast as

$$e^{-\phi} = 4\sqrt{2}\, e^{K^K/2}\, \frac{\operatorname{Im} W}{Q_0}\,, \tag{4.28}$$

which determines the dilaton once the complex and Kähler moduli have been solved for.

Before turning to the Kähler moduli, we derive a useful consequence of the complex structure equations. Multiplying the equations (4.24) by $\operatorname{Re}(CZ_k)$ and $\operatorname{Re}(Cg_\lambda)$, respectively, summing over k and λ and taking the difference, we find using the identity (4.17) that

$$-iW = \sum_\lambda q_\lambda \operatorname{Re}(Cg_\lambda) - \sum_k p_k \operatorname{Re}(CZ_k) \equiv \frac{1}{2} \operatorname{Im} W^Q\,. \tag{4.29}$$

Hence when the complex structure moduli satisfy their equations of motion, the vacuum value of the superpotential can be written in terms of the Kähler moduli only:

$$W(t_a, N_k, T_\lambda) = -i \operatorname{Im} W^K(t_a)\,. \tag{4.30}$$

4.3.2 Kähler equations

The relation (4.30) allows us to decouple the Kähler sector. Using (4.30), the equations $D_{t_a}W = 0$ become

$$\partial_{t_a} W^K - i\partial_{t_a} K^K \operatorname{Im} W^K = 0\,. \tag{4.31}$$

Hence we can consider these equations entirely independently from the hypermultiplet moduli and H_3 fluxes.

In the analysis that follows we will assume nonvanishing m_0. It is straightforward to show that for $m_0 = 0$, one must either have $m_a = e_a = 0$ as well, and the Kähler moduli are then all unfixed, or the v_a are driven to zero, far from the large-volume region.

Again it is useful first to consider the imaginary parts of the equations. Since K^K depends only on $v_a \equiv \operatorname{Im} t_a$, the second term in (4.31) is real. Thus we find

$$\operatorname{Im} \partial_{t_a} W^K = \kappa_{abc} v_b (m_c - m_0\, b_c) = 0\,, \tag{4.32}$$

(recall $b_c = \operatorname{Re} t_c$). The regularity of the moduli space metric implies there is always some κ_{abc} that is nonzero for any given c; assuming the 2-cycle volumes v_b do not vanish, as will be the case for example in a geometrical limit, one finds for all c:

$$b_c = \frac{m_c}{m_0}\,. \tag{4.33}$$

We see that unlike the case of the complex structure, for the Kähler moduli the axions are generically all fixed. As we will discuss further in section 6, this can be understood as arising from the fact that the Kähler sector has twice as many fluxes per real modulus as the complex structure (m^a, e_a for the Kähler sector as opposed to p_q, q_λ for the complex structure sector).

Consider now the real part of the equations (4.31). Using the axion solution (4.33), one can write these equations as

$$\begin{aligned}\left(3m_0^2\kappa_{abc}v_bv_c + 4e_am_0 + 2\kappa_{abc}m_bm_c\right)(\kappa_{def}v_dv_ev_f)+ \\ +(\kappa_{abc}v_bv_c)(6m_0e_dv_d + 3\kappa_{def}m_dm_ev_f) = 0\,.\end{aligned} \tag{4.34}$$

Multiplying by v_a and summing over a, we have

$$3m_0^2(\kappa_{abc}v_av_bv_c) + 10m_0e_av_a + 5\kappa_{abc}m_am_bv_c = 0\,. \tag{4.35}$$

Substituting this back into (4.34) and cancelling an overall factor, one finds for each a,

$$3m_0^2\kappa_{abc}v_bv_c + 10m_0e_a + 5\kappa_{abc}m_bm_c = 0\,. \tag{4.36}$$

These $h_-^{1,1}$ simple quadratic equations for the $h_-^{1,1}$ moduli v_a are the final result; we have as many equations as unknowns and expect all the moduli to be frozen. Let us discuss a few properties of these equations.

A key feature of (4.36) is that Kähler moduli are only coupled to other Kähler moduli with which they have a nonvanishing triple intersection; this is not obvious from the original equations (4.31). In studying our example $T^6/\mathbb{Z}_3^2$ in section 5, we shall see that this justifies treating every blow-up mode independently from the other blow-ups, as well as from the untwisted moduli, even when the latter are not taken to be much larger than the blow-ups.

Using (4.35), one can show that

$$W = -i\,\mathrm{Im}\,W^K = \frac{2i}{15}m_0\kappa_{abc}v_av_bv_c\,. \tag{4.37}$$

From this we learn that $W = 0$ cannot occur for this class of vacua without the overall volume $\int J\wedge J\wedge J$ vanishing. This justified the assumption of $\mathrm{Im}\,W \neq 0$ we made in analyzing the complex structure moduli.[6]

Using (4.37) one can solve for the dilaton using (4.28). One can see from (4.36), (4.37) that under a flip of the sign of all RR fluxes, $W \to -W$. Thus to preserve the physically correct sign for the dilaton (4.28), one must flip the signs of the H_3 fluxes as well. (The periods $\mathrm{Im}\,g_k$ and $\mathrm{Im}\,Z_\lambda$ have definite sign fixed by the sign of Ω, which in turn is fixed as it calibrates the special lagrangian submanifolds on which the O6s are wrapped.) It is familiar from studying type IIB vacua that flipping signs of the RR fluxes without doing likewise for the NSNS fluxes leads to a solution with unphysical dilaton, an indication that the solution preserves the opposite sign of supersymmetry; the sign of the tadpole from the fluxes has been flipped, and in this case, those fluxes are consistent with an anti-O6 background instead of an O6 background.

Let us summarize the equations determining the supersymmetric vacua.

Kähler moduli b_a, v_a:

$$b_a = \frac{m_a}{m_0}\,, \qquad 3m_0^2\kappa_{abc}v_bv_c + 10m_0e_a + 5\kappa_{abc}m_bm_c = 0\,. \tag{4.38}$$

[6] A compactification with $m_0 \neq 0$ and all other fluxes vanishing (and no orientifold) was studied in [35], where it was found that the solution is forced to $\int J\wedge J\wedge J = 0$, consistent with (4.35).

Complex structure moduli $\operatorname{Im} g_k, \operatorname{Re} Z_\lambda$:

$$\begin{aligned} \operatorname{Im} g_k &= 0 \text{ for } p_k = 0\,, \qquad \operatorname{Im} Z_\lambda = 0 \text{ for } q_\lambda = 0\,, \\ e^{-K^{\rm cs}/2}\,\frac{p_{k_1}}{\operatorname{Im} g_{k_1}} &= e^{-K^{\rm cs}/2}\,\frac{p_{k_2}}{\operatorname{Im} g_{k_2}} = \cdots = e^{-K^{\rm cs}/2}\,\frac{q_{\lambda_1}}{\operatorname{Im} Z_{\lambda_1}} = \cdots \equiv Q_0\,, \quad \text{for all } p_{k_i}, q_{\lambda_j} \neq 0\,. \end{aligned} \tag{4.39}$$

Dilaton ϕ:

$$e^{-\phi} = \frac{4\sqrt{2}}{5\sqrt{3}}\,\frac{m_0}{Q_0}(\kappa_{abc}v^a v^b v^c)^{1/2}\,. \tag{4.40}$$

One axion $q_\lambda\tilde{\xi}_\lambda - p_k\xi_k$:

$$p_k\xi_k - q_\lambda\tilde{\xi}_\lambda = \operatorname{Re} W^K = e_0 + \frac{e_a m_a}{m_0} + \frac{\kappa_{abc}m_a m_b m_c}{3m_0^2}\,. \tag{4.41}$$

These equations assume $v_a \neq 0$ and $\kappa_{abc}v_a v_b v_c \neq 0$.

Note that in general we need m_0 and at least one p_k or q_λ to be nonzero for a stabilized vacuum; if either condition fails, all fluxes must vanish and the moduli go unstabilized. The minimum set of fluxes required to stabilize all geometric moduli is m_0, one p_k or q_λ (satisfying the orientifold tadpole) and one e_a or m_a for each Kähler modulus.

It will generally be true that some fluxes will lead to solutions of (4.38) lying outside the geometric regime; for example in section 5 we will see that for the $T^6/\mathbb{Z}_3^2$ orientifold some fluxes imply some $v^a < 0$. In this regime we expect not just the Kähler potential K, but also the superpotential W, to receive α' corrections, and hence the result cannot be trusted.

When some of the p_k or q_λ vanish, one ends up with either $g_k = 0$ or $Z_\lambda = 0$. The vanishing of a linear combination of periods does not a priori mean that a 3-cycle has collapsed; such vanishing occurs at a dense set of points in moduli space, while the actual discriminant locus is of codimension one.[7] In the rare case where such a 3-cycle has collapsed, one might worry about being driven to a singularity on moduli space where new fields become light. However in type IIA string theory, the complex structure moduli space is embedded within the quaternionic hypermultiplet moduli space, within which singularities have codimension four or higher. Even after the orientifold projection, since the surviving axion partners of the complex moduli are in general unfixed by the fluxes, one need not end up at a singular point; landing at the singular point in moduli space will require a tuning of the axion vevs.

4.4 Gauge redundancies

There are in general modular group transformations, acting both on the moduli and on the fluxes, that relate equivalent vacua. In particular, it is evident that there are two kinds of modular transformations of infinite order, those that shift the complex structure axions ξ_k, $\tilde{\xi}_\lambda$ by one, and those that shift the Kähler axions b_a likewise. Here we derive the action of these transformations on the fluxes; we do so using the fact that the $DW = 0$ equations must transform covariantly, so solutions are mapped to other solutions. We also heuristically describe the nature of the transformation in a T-dual type IIB picture.

[7] We thank F. Denef for reminding us of this fact.

Consider first the Kähler axions. From (4.33), is is obvious that a shift of the axion $t_a \to t_a + 1$ corresponds to a shift of $m_a \to m_a + m_0$. Assuming m_0 is fixed, the first term of (4.35) is unchanged, thus determining the action on e_a. Finally the invariance of $\mathrm{Re}\, W^K$ can be used to fix the transformation of e_0. The result is, for integers u_a,

$$\begin{aligned} t_a &\to t_a + u_a\,, \\ m_0 &\to m_0\,, \quad m_a \to m_a + u_a m_0\,, \quad e_a \to e_a - \kappa_{abc} m_b u_c\,, \quad e_0 \to e_0 - e_a u_a\,. \end{aligned} \tag{4.42}$$

This transformation can be regarded as the T-dual of a geometric transformation. Consider the $(T^6)/\mathbb{Z}_3^2$ model and a shift of $t \to t+1$ for one of the tori, which is a shift in B_2 integrated over that T^2. Taking a single T-duality in this T^2, the shift of t is mapped to trivial shift of the complex structure of the dual torus, while the RR fields are mapped into modes of F_1, F_3 and F_5, which are mixed amongst each other by this geometrical shift in precisely the way specifying (4.42).

Next consider shifts of the complex structure axions. Consider for example $\xi_k \to \xi_k + 1$, which requires $\mathrm{Re}\, W^K \to \mathrm{Re}\, W^K + p_k$; this can be accomplished with a shift of e_0 alone. In general we find

$$\xi_k \to \xi_k + U_k\,, \quad \tilde{\xi}_\lambda \to \tilde{\xi}_\lambda + V_\lambda\,, \quad e_0 \to e_0 + p_k U_k - q_\lambda V_\lambda\,, \tag{4.43}$$

for integers U_k, V_λ. When only one component of H_3 is turned on, this transformation can be understood as the mirror of type IIB $SL(2,Z)$ shifts; three T-dualities take H_3 and F_6 to type IIB H_3 and F_3 polarized along the same directions, which are then mixed by an $SL(2,Z)$ transformation.

5. Application to $T^6/\mathbb{Z}_3^2$ model

We now apply the results of the previous section to the specific case of our $T^6/\mathbb{Z}^3$ model, searching for solutions in the limit where all volumes are sufficiently large that we can neglect α' corrections.

We shall denote the F_2 and F_4 fluxes associated to the untwisted cycles by m_i and e_i, $i = 1, 2, 3$ while those on the blow-ups are n_A and f_A, $A = 1 \ldots 9$; the corresponding moduli are the untwisted Kähler modes t_i and the blow-up Kähler modes t_{B_A}. In the hypermultiplet sector, $h^{2,1} = 0$ and we have only the index $k = 0$ and no λ indices; the unique flux is $p_{k=0} \equiv p$, and the moduli are just the dilaton ϕ and its axionic partner $\xi_{k=0} \equiv \xi$.

5.1 General solution

We first consider the Kähler sector. The nonzero elements of the intersection form are $\kappa_{123} = \kappa$ and $\kappa_{AAA} = \beta$, and consequently we can solve for each of the blow-up modes independently of the untwisted moduli and of the other blow-up modes.[8] Considering first

[8] The values of κ and β can be found by a simple modification of the results in [19], where the intersection form of $T^6/\mathbb{Z}_3$ was computed. The result (correcting a minor error in [19] and accounting for the further free $\mathbb{Z}_3$ action) is that $\kappa = 81$ and $\beta = 9$, but we will continue the discussion in terms of variable κ, β.

the untwisted moduli, the axions are fixed as (4.33),

$$b_i = \mathrm{Re}\, t_i = \frac{m_i}{m_0}\,, \tag{5.1}$$

while for the volumes $v_i = \mathrm{Im}\, t_i$ we find the equations (4.36)

$$6m_0^2\kappa v_2 v_3 + 10m_0 e_1 + 10\kappa m_2 m_3 = 0\,, \tag{5.2}$$
$$6m_0^2\kappa v_1 v_3 + 10m_0 e_2 + 10\kappa m_1 m_3 = 0\,, \tag{5.3}$$
$$6m_0^2\kappa v_1 v_2 + 10m_0 e_3 + 10\kappa m_1 m_2 = 0\,. \tag{5.4}$$

The solution to this system is

$$v_i = \frac{1}{|\hat{e}_i|}\sqrt{\frac{-5\hat{e}_1\hat{e}_2\hat{e}_3}{3m_0\kappa}}\,, \tag{5.5}$$

where we have defined the shifted flux $\hat{e}_i$ invariant under shifts of t_i (4.42),

$$\hat{e}_i \equiv e_i + \frac{\kappa m_j m_k}{m_0}\,, \tag{5.6}$$

where j and k are simply the two values other than i.

For each of the blow-up modes, the volumes v_{B_A} satisfy

$$3m_0^2\beta v_{B_A}^2 + 10m_0 f_A + 5\beta n_A^2 = 0\,, \tag{5.7}$$

with no sum over A. The solution for the complex blow-up moduli is then[9]

$$t_{B_A} = \frac{n_A}{m_0} - i\sqrt{\frac{-10\hat{f}_A}{3\beta m_0}}\,. \tag{5.8}$$

where again we defined an invariant shifted flux $\hat{f}_A$,

$$\hat{f}_A \equiv f_A + \frac{\beta n_A^2}{2m_0}\,. \tag{5.9}$$

There are no complex structure moduli, so only the dilaton and its axion ξ remain. Using the dilaton equation (4.28), and the results $\mathrm{Im}\, g_0 = -1/\sqrt{2}$, $K^{\mathrm{cs}} = 0$, $e^{K^K} = 3/(4\kappa_{abc}v_a v_b v_c)$, we find

$$e^{-\phi} = -\frac{4\sqrt{3}}{15}\frac{m_0}{p}(\kappa_{abc}v_a v_b v_c)^{1/2}\,, \tag{5.10}$$

where the total volume (proportional to the 4D coupling e^{-D}) is given by

$$\kappa_{abc}v_a v_b v_c = -\frac{15p}{2\sqrt{2}m_0}e^{-D} = \frac{10}{|m_0|}\sqrt{\frac{-5\hat{e}_1\hat{e}_2\hat{e}_3}{3m_0\kappa}} + \beta\sum_A\left(\frac{-10\hat{f}_A}{3\beta m_0}\right)^{3/2}\,, \tag{5.11}$$

where we have used the fact, discussed in the next subsection, that $\mathrm{sgn}(m_0\hat{e}_1\hat{e}_2\hat{e}_3) < 0$ must hold. Finally the axion ξ is fixed as (4.26)

$$\xi = \frac{\mathrm{Re}\, W^K}{p} = \frac{1}{p}\left(e_0 + \frac{e_i m_i + f_A n_A}{m_0} + \frac{6\kappa m_1 m_2 m_3 + \beta\sum_A n_A^3}{3m_0^2}\right)\,. \tag{5.12}$$

[9] Note that to stay within the Kähler cone, one should choose the solution with $\mathrm{Im}\, t_{B_A} < 0$; this unusual convention arises because the self-intersection of the resolving $\mathbb{P}^2$ of a $\mathbb{C}^3/\mathbb{Z}_3$ singularity is -3 times an actual curve.

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5.2 Regime of validity and agreement with 10D analysis

This solution will be valid as long as the volumes v_i, v_{B_A} are sufficiently large that α' corrections can be neglected, and the string coupling is small enough that quantum corrections can be neglected. One can see from (5.5) and (5.8) that the volumes are large whenever

$$|\hat{e}_i| \gg |m_0|\,, \qquad |\hat{f}_A| \gg |m_0|\,. \tag{5.13}$$

Moreover, to remain within the Kähler cone, we must ensure the untwisted volumes are sufficiently larger than the blow-ups, requiring

$$|\hat{e}_i| \gg |\hat{f}_A| \gg |m_0|\,. \tag{5.14}$$

Because the four-form and two-form fluxes are not constrained by the tadpole, we are free to scale them to be as large as we wish. Thus we can always choose some fluxes obeying (5.14) that provide a geometric solution.

When the hierarchy (5.14) is obeyed, the behavior of physical quantities is dominated by the F_4 flux for the non-blow up cycles. Let us again take $\hat{e}_i \sim \bar{e}$; we have shown that the Kähler parameters scale as $v_i \sim \bar{e}^{1/2}$, becoming large with large $\bar{e}$. Then in addition to the overall volume becoming big, the ten- and four-dimensional string couplings become small in this limit:

$$\mathrm{vol} \sim \bar{e}^{3/2}\,, \qquad e^{\phi} \sim \bar{e}^{-3/4}\,, \qquad e^{D} \sim \bar{e}^{-3/2}\,, \tag{5.15}$$

suppressing quantum corrections.

One may be concerned that even though the volumes are much larger than α', higher derivative corrections to the 10D lagrangian may nonetheless become relevant, because the flux parameter $\bar{e}$ will increase the coefficient of certain terms as it grows large. We can estimate the size of higher order corrections involving powers of $|F_4|^2$ as follows.

First, two powers of F_4 give an explicit $\bar{e}^2$ scaling. Next there are 4 factors of the inverse metric in contracting the indices of the form fields, which provides a factor of $R^{-8} \sim \bar{e}^{-2}$. Finally, it is a famous fact that RR vertex operators are accompanied by an extra factor of g_s, yielding an additional power of $\bar{e}^{-3/2}$.

Assembling all of the ingredients, we see that relative to the leading term in the 10D lagrangian, terms with additional powers of $|F_4|^2$ are suppressed by an expansion parameter $\lambda \sim \bar{e}^{-3/2}$. Therefore, in the large $\bar{e}$ limit, we expect corrections from both the α' and g_s expansions to be parametrically suppressed. The existence of these SUSY vacua is therefore robust against any known corrections.

The scalings (5.15) are the same as those found in the 10D analysis; in fact, in the limit (5.14), where the fluxes on the non-blow-up cycles dominate the string coupling, the solution (5.5), (5.10) agrees precisely with (3.31), and the blow-up volume (5.8) agrees qualitatively with the estimate (3.45), with the replacement $\hat{e}_i \to e_i$, $\hat{f}_A \to f_A$ to reflect the special case $m_i = n_A = 0$. There is one subtlety: the signs in the 4D analysis are more constrained than those in the 10D analysis. In particular, although both analyses agree that a solution requires

$$\mathrm{sgn}\,(m_0 p) < 0\,, \tag{5.16}$$

the 4D supersymmetric equations also imply a constraint on the signs of the F_4 fluxes,

$$\mathrm{sgn}\,(m_0\hat{e}_1\hat{e}_2\hat{e}_3) < 0\,, \qquad \mathrm{sgn}(m_0\hat{f}_A) < 0\,, \tag{5.17}$$

as well as the condition

$$\hat{e}_i v_1 = \hat{e}_2 v_2 = \hat{e}_3 v_3\,, \tag{5.18}$$

requiring the signs of the $\hat{e}_i$ all to coincide:

$$\mathrm{sgn}\,\hat{e}_1 = \mathrm{sgn}\,\hat{e}_2 = \mathrm{sgn}\,\hat{e}_3\,, \tag{5.19}$$

in order for the v_i to all be positive and hence in the large-volume region. The results of section 3, however, imply that even if the signs of the $\hat{e}_i$ are not aligned, there is still a solution at positive v_i, necessarily non-supersymmetric as it violates (5.19), but apparently lacking in instabilities. The nature of these extra solutions, and the exact location of the supersymmetric vacuum in the small volume region, we leave for future work.

Since $W \neq 0$, these supersymmetric vacua are anti-de Sitter. Hence another interesting quantity to consider is the 4D cosmological constant Λ. One finds

$$\Lambda = -3e^{K^K+K^Q}|W|^2 \sim \bar{e}^{-9/2}\,. \tag{5.20}$$

It is natural to ask whether the vacuum can be treated as effectively four-dimensional: this will be the case if the Hubble scale H, defined as

$$H^2 = \frac{\Lambda}{M_P^2}\,, \tag{5.21}$$

with M_P^2 the four-dimensional Planck scale, is less than the Kaluza-Klein scale $1/R$. Using the four-dimensional Einstein frame where $M_P \sim \bar{e}^0$, we calculate that $R^2 \sim \bar{e}^{7/2}$, leading to the result

$$(HR)^2 \sim \frac{1}{\bar{e}}\,. \tag{5.22}$$

Hence there is a parametric hierarchy between the AdS radius and the Kaluza-Klein scale, and treating the vacuum with four-dimensional effective theory makes sense. This is to be contrasted with the case of the Freund-Rubin vacua which feature most prominently in examples of the AdS/CFT correspondence, where the KK scale and the scale of the cosmological constant are the same, and the background is not effectively four-dimensional.

Hence we have demonstrated for the $T^6/\mathbb{Z}_3^2$ orientifold the existence of parametrically tunable large volume, weak coupling flux vacua with a valid four-dimensional description and all moduli stabilized.

6. Rudimentary IIA vacuum statistics

> "To understand God's thoughts we must study statistics, for these are the measure of His purpose."
>
> — *Florence Nightingale*

It would be interesting to do a thorough analysis of the statistics of type IIA flux vacua; related M-theory models were recently studied in [32]. Here, we make a modest contribution by analyzing the statistics in the simplest toy model, a fictitious rigid Calabi-Yau space with a single Kähler modulus t and no complex structure moduli, with fluxes m_0, m, e, e_0 and p. Taking $\kappa = 1$ we have

$$W^K = e_0 + e\,t + \frac{1}{2} m\,t^2 - \frac{m_0}{6}\,t^3 \,. \tag{6.1}$$

This example may be viewed as somewhat analogous to the type IIB rigid Calabi-Yau toy model studied in [11].

The solution for the Kähler modulus t is identical to that of (5.8) for a single blow-up mode,[10]

$$t = \frac{m}{m_0} + i\sqrt{\frac{-10\hat{e}}{3m_0}}\,, \qquad \hat{e} \equiv e + \frac{m^2}{2m_0}\,, \tag{6.2}$$

while the dilaton and axion have the solutions

$$e^{-\phi} \sim \frac{m_0}{p}(\mathrm{Im}\,t)^3\,, \quad \xi = \frac{1}{p}(e_0 + \frac{em}{m_0} + \frac{m^3}{3m_0^2})\,. \tag{6.3}$$

While we have chosen to analyze this case for simplicity, it is easy to see that the solutions for $T^6/\mathbb{Z}_3^2$ are virtually identical when $e_1 = e_2 = e_3$, $m_1 = m_2 = m_3$. In fact, because of the simple form of the Kähler equations (4.36), all solutions for Kähler moduli in the geometric regime will have the general form (6.2). Hence we are able to capture the essential behavior of all flux-frozen Kähler moduli by studying (6.2).

The tadpole condition in general requires

$$0 < -m_0 p \leq N\,, \tag{6.4}$$

where N is the magnitude of the negative D6 charge induced by O6 planes wrapping the fixed point locus of the anti-holomorphic involution σ. In the cases that the inequality is not saturated, one can compensate by including D6 branes. In simple examples (including our explicit case) N is $\mathcal{O}(1)$, and we shall assume some fixed (though possibly large) N in the following analysis (i.e. we will *not* work in an $N \to \infty$ limit). In a more general model with some complex structure moduli, there will be a tadpole like (6.4) for each p or q, limiting the possible values of all the NSNS fluxes.

By varying flux integers, it appears that one can easily obtain a denumerably infinite number of vacua. However, the naive analysis significantly over-counts solutions, as there are modular symmetries that relate vacua with different values of these parameters. While we will only do approximate statistics of various asymptotics in an appropriate large flux limit, to avoid making a serious error we need to gauge fix the modular group generators of infinite order. These are the two symmetries discussed in section (4.4): integer shifts of $\mathrm{Re}\,t$ and of ξ, which we account for as follows.

[10] Unlike in (5.8), here we have chosen the positive root, assuming v is the volume of a physical curve.

Rather than restricting the value of the moduli with the gauge symmetries, we choose instead to restrict possible choices of the fluxes. Using the shift symmetry (4.42) of the Kähler axions, we can restrict

$$0 \leq m < |m_0| \,, \tag{6.5}$$

leaving us with $|m_0|$ inequivalent choices of the flux m for fixed m_0; in more complicated models there will be one such symmetry for each m_a, permitting them all to be restricted in this way. We can estimate the number of such choices as

$$\sum_{M=1}^{N} \sum_{m_0|M} |m_0| = \sum_{M=1}^{N} \sigma(M) \sim \frac{\pi^2}{12} N^2 \,. \tag{6.6}$$

We can then fix the shift symmetry (4.43) of the axion ξ by restricting e_0:

$$0 \leq e_0 < p \,, \tag{6.7}$$

giving us p possible values. In fact, this is not independent of the previous discussion: for a given partition of $M \leq N$ into $m_0 p$, one gets m_0 choices for m and p choices for e_0, so we should replace (6.6) by the slightly more elaborate

$$\sum_{M=1}^{N} \sum_{m_0|M} M = \sum_{M=1}^{N} M d(M) \sim N^2 \log N \,. \tag{6.8}$$

Notice that in models with multiple ξ axions, further gauge fixing beyond the restriction (6.7) will be necessary.

At this point we can see that for a given orientifold, m_0 and p are constrained to take a finite number of values, and the degeneracy of vacua from varying m and e_0 is given by (6.8); almost all the fluxes have been restricted to finitely many values. However, we are still free to vary e while satisfying all tadpole conditions, and we have no more infinite order modular symmetries to reduce the space of choices to a finite set. Furthermore, we see from (6.2), (6.3) and that if we are concerned with the large e asymptotics of the solutions (as we will be), then the allowed variations of m_0, m at fixed N will have a minor effect. In the explicit example, for instance, $N = \mathcal{O}(1)$ and the additional degeneracy factors discussed here are completely irrelevant for understanding the distribution of vacua at large e.

The upshot of this discussion is that, in this gauge fixing, the statistics are dominated by the large e vacua and we shall focus henceforth on their properties.

6.1 Statistics and general comments

Although the number of vacua diverges, there are still interesting statistical questions that one can ask. The well-posed questions are questions like: how many vacua exist below a given volume? How many vacua exist above a given $|\Lambda|$?

It is easy to answer these questions using the scaling results of the previous subsection; essentially all that matters is the large e behavior, since this is where an infinite number of vacua lie, with their properties dominated by e. The finite range of values of the other fluxes then only contributes to very fine structure in the space of vacua.

Using the fact that the length scale of the compact space in string frame scales as $R \sim e^{1/4}$, we see that (at least for sufficiently large R^*) the number of vacua with $R \leq R^*$ scales like $(R^*)^4$:

$$\mathcal{N}(R \leq R^*) \sim (R^*)^4\,. \tag{6.9}$$

In previous cases, Calabi-Yau flux vacua have had distributions governed by the volume form on the appropriate moduli space; we note here that (6.9) does *not* conform to a distribution on the Kähler moduli space governed by the volume form arising from (4.6).

For the cosmological constant, using $\Lambda \sim e^{-9/2}$, one has

$$\mathcal{N}(|\Lambda| \geq |\Lambda^*|) \sim (|\Lambda^*|)^{-2/9}\,. \tag{6.10}$$

In other words, the number distributions of vacua (without any assumptions about a cosmological measure) favor large volume and small cosmological constant, in this supersymmetric ensemble. Note that one should not trust the distribution (6.10) at large $|\Lambda|$ because our approximations are invalid at small e. Hence the slow power of the decay in this limit should not cause concern; any structure in the distribution of vacua at *large* $|\Lambda|$ is not trustworthy.

Given the large amount of recent work on characterizing the string landscape, it seems worthwhile to make some comments about the similarities and differences of our results to those obtained in other ensembles. Firstly, we should emphasize that the divergence of the number of SUSY vacua may not be particularly disastrous. A mild cut on the acceptable volume of the extra dimensions will render the number of vacua finite. On the other hand, one can legitimately worry that the conclusions of any statistical argument will be dominated by the precise choice of the cut-off criterion, since the regulated distribution is dominated by vacua with volumes close to the cut-off.

Secondly, we should comment that our statistical results are qualitatively rather similar to those obtained in [32] for AdS_4 Freund-Rubin vacua of M-theory. A promising difference between these two sets of vacua is the parametric ratio we obtain between the Hubble scale and the scale of the internal dimensions, which is generally absent in Freund-Rubin vacua.

6.2 Comparison to other ensembles of vacua

By far the most well-studied example of flux vacua in string theory is the set of type IIB vacua with the Calabi-Yau complex structure moduli and dilaton stabilized by H_3 and F_3 fluxes. In addition, recently there has been discussion [32] of statistics for moduli-stabilized flux vacua in compactifications of M-theory on manifolds of G_2 holonomy. It is naturally interesting to compare the ensembles to the IIA system we study.

In principle any vacuum of string theory can be described in an alternate duality frame, and so the vacua we describe should be expressible in the language of type IIB string theory via mirror symmetry, or of M-theory by relating the string coupling to the M-theory circle.

However, our vacua need not admit a description as a flux compactification in the dual language. In fact, generically some parameters associated to fluxes will be mapped to geometric torsions, which are considerably more difficult to characterize; an understanding of them on the same level as fluxes has yet to be obtained [17, 36]. Furthermore, the global properties of the dual-spaces may even be nongeometric [37]. Hence by studying flux compactifications of a given theory, without torsions, we are choosing a different "slice" of all possible compactifications than we would obtain by studying the flux compactifications of another theory.

So while by studying torsions as well as fluxes we could in principle see that two dual descriptions of string theory have the same vacuum statistics,[11] different ensembles of flux compactifications alone will not in general agree. Hence it is interesting to compare them.

We will take a small step in this direction by comparing the ratio of available fluxes to moduli in four different ensembles: Type IIA Kähler, type IIA complex structure, type IIB complex structure, and M-theory on G_2. Define the ratio

$$\eta \equiv \frac{\#\text{ real flux parameters}}{\#\text{ real moduli}}\,. \tag{6.11}$$

In general the larger the η parameter is for a given ensemble, the more moduli can be fixed, and the less "friendly" the distribution will be (in the language of [34]); similar observations have been put forward in [10, 32].

The simplest ensemble is the case of M-theory on a G_2 manifold [5, 32]. There are b_3 complex moduli z_i, and b_3 G_4 fluxes N^i, as well as the complex Chern-Simons invariant $c_1 + ic_2$. Hence we have $\eta_{G_2} = (b_3+2)/(2b_3) \sim 1/2$. The superpotential has the structure

$$W_{G_2} = c_1 + ic_2 + z_i N^i\,. \tag{6.12}$$

In this ensemble, nonzero c_2 is required for solutions at finite volumes $s_i = \operatorname{Im} z_i$, and only a single linear combination of the axions $\operatorname{Re} z_i$ are fixed. We may understand this heuristically as since $\eta \sim 1/2$, there are only as many fluxes as there are volume parameters s_i, and consequently the axions are left unfixed.

Consider next the type IIA Kähler sector studied in this paper; since it can be completely decoupled from the other moduli, it makes sense to consider it independently. There are $h^{1,1}_-$ complex moduli t_a, and $2h^{1,1}_- + 2$ RR fluxes; hence we find $\eta_{IIA,K} = (2h^{1,1}_- + 2)/(2h^{1,1}_-) \sim 1$. The superpotential

$$W_{IIA,K} = e_0 + t_a e_a + \frac{1}{2}\kappa_{abc} m_a t_b t_c - \frac{m_0}{6}\kappa_{abc} t_a t_b t_c\,, \tag{6.13}$$

is structurally a generalization of (6.12), with the fluxes m_a and m_0 generating quadratic and cubic terms. With this doubling of the number of fluxes, one finds that the axions as well as the geometric moduli are stabilized.

[11] This is not guaranteed, however, especially if one only computes the statistics for those vacua which are weakly coupled in the respective corners (which may be the sensible thing to do).

Hence one sees how in passing from an M-theory description to a IIA description, additional parameters that were described in terms of the geometry have become available as fluxes, and the increase in the number of fluxes allows all moduli to be stabilized. Note that (6.13) has no precise analog of c_2 in (6.12), the Chern-Simons invariant introduced by Acharya [5] to achieve nontrivial moduli stabilization, but m_0 plays a very similar role.

Next consider the other ensemble in type IIA compactifications, that of the complex structure moduli and dilaton. There are $h^{2,1}+1$ complex moduli, and in addition to the $h^{2,1}+1$ H_3 fluxes, one requires the complex number $\mathrm{Im}\, W^K$ from the Kähler sector as input. Hence we have $\eta_{IIA,c} = (h^{2,1}+3)/(2(h^{2,1}+1)) \sim 1/2$. Since one has $\eta_{IIA,c} = \eta_{G_2}$, one might expect a similar story, and this is what we found: as in the G_2 case, the geometric moduli are frozen, but the axions are not except for a single linear combination. Hence we see that although the G_2 superpotential superficially resembles the IIA Kähler case more strongly (they are both simple polynomials in the moduli), its behavior is much more like that of the IIA complex structure case, and this similarity can be traced to their having the same value of η.

The final familiar ensemble is that of type IIB, with imaginary self-dual fluxes stabilizing the complex structure moduli and dilaton. In this case there $h^{2,1}+1$ complex moduli, but $4(h^{2,1}+1)$ fluxes; hence $\eta_{IIB} = 4(h^{2,1}+1)/(2(h^{2,1}+1)) = 2$. This is the largest number of fluxes per modulus of all these ensembles; one may think of starting with the type IIA complex structure ensemble and doubling the fluxes, as F_3 contributes as well as H_3, effectively complexifying the flux. (Of course, since in IIB the RR fluxes as well as the NSNS fluxes go into stabilizing the complex structure moduli, there are none left to stabilize the Kähler moduli.) Not only are all moduli frozen, but additional choices are left over, allowing a broader, less "friendly" distribution.

Thinking ahead, the inclusion of torsions as well as fluxes will naturally cause the suitable generalization of the η parameter to grow. Hence when one considers all the discrete choices that characterize these generalized flux compactifications, stabilization of all moduli becomes increasingly easy, and distributions become less and less "friendly". It is quite reasonable to expect that a generic example of such a generalized flux compactification would stabilize all moduli, regardless of the particular string theory considered.

7. Conclusions

The main striking features of the class of models described in this paper are their simplicity, and the appearance of a parameter which yields power-law parametric control. In the supersymmetric vacua of the IIB theory where all moduli are stabilized [1], the control parameter only grows logarithmically with a tuning parameter; hence, while one can make controlled constructions, it requires precise tuning in a large space of flux vacua. Here, in contrast, the radii and couplings fall into a controlled regime as a power of the F_4 flux. This gives these models special appeal as a setting to do controlled studies of fully stabilized string vacua. It also hints that finding dual CFTs, which is a difficult problem for the AdS models of [1], may be considerably simpler here; the large flux limit may admit a simple dual description.

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It would be worthwhile to find proposals for perturbing these vacua by small positive energies to yield controlled de Sitter models, perhaps along the lines of similar proposals in the type IIB theory [1, 38]. In addition, the inclusion of perturbative corrections to K, worldsheet instantons (whose effects should be computable by using mirror symmetry and co-opting the appropriate type IIB computations of prepotentials), and Euclidean D2 instanton effects, could add very interesting features to these potentials; in the analogous $\mathcal{N} = 2$ setting quantum corrections certainly do seem to play an important role [14]. At least the worldsheet instanton effects should be something that one can incorporate at the level of statistical analyses. There has also been great progress in constructing realistic brane world models in flux backgrounds [39] and in using the fluxes to freeze the open string moduli [40, 41] and induce soft supersymmetry breaking terms [42]; it would be interesting to combine these ingredients in the setting suggested here.

Finally, it would be interesting to see if there is a direct relation between our IIA constructions and some topological field theory construction, which could provide an analogue of the Hartle-Hawking wavefunction [43] for these vacua — such a construction has been obtained for some simple classes of Freund-Rubin vacua in [44]. We note here that any naive application of the Hartle-Hawking wavefunction to obtain a measure on this set of vacua will suffer from the same problem of cut-off dominance mentioned in section 6.1 in the context of statistical arguments. Without imposing a cut-off on the four-form flux, the wavefunction will be badly non-normalizable (as it is for the analogous black hole problem in [44], if one does not impose a cut-off on the allowed charges). Imposing a cut-off, one will find that the wavefunction is peaked at the cut-off; this is the analogue of the cut-off domination problem for statistical arguments. One proposal to fix this problem in the more physical case of de Sitter vacua has been described in the papers [45], which also provide references to further critical discussion in the quantum cosmology literature. At any rate it is clearly a worthwhile and ambitious goal to find a good measure on the space of vacua. Success will require both a detailed knowledge of the structure of the space of vacua, and significant new insights into early universe cosmology in string theory.

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A. IIA Chern-Simons term in presence of fluxes

In this appendix we consider the Chern-Simons term of IIA supergravity in the presence of topological fluxes. We only consider the massless IIA theory here, which can be derived through dimensional reduction from M-theory, and give an elementary derivation of that subset of the full set of Chern-Simons terms that plays a role in the analysis of this paper. A full treatment of the Chern-Simons terms of type IIA string theory is rather subtle and requires dealing properly with flux self-duality, anomaly cancellation and the classification of fluxes in K-theory [24], and leads to additional contributions involving curvature forms and an overall sign for the exponentiated action; we neglect such terms here.

The Chern-Simons term of IIA supergravity is well known to be given in the absence of topological fluxes by

$$S_{\rm CS} = \frac{1}{2\kappa_{10}^2} \int H_3 \wedge C_3 \wedge F_4 \,. \tag{A.1}$$

In the absence of topological fluxes, this Chern-Simons term can be integrated by parts to give

$$S_{\rm CS} = -\frac{1}{2\kappa_{10}^2} \int B_2 \wedge F_4 \wedge F_4 \,. \tag{A.2}$$

These two forms of the Chern-Simons contribution to the action are generally used interchangeably. Note, however, that in the presence of a topological flux $H_3^{\rm bg}$ or $F_4^{\rm bg}$ there is a subtlety. When such a flux is present, the boundary terms $\int_\partial B_2 \wedge C_3 \wedge F_4$ arising from the integration by parts may not vanish, due to a large gauge transformation which relates the forms B_2, C_3 at two images of the same boundary. Thus, the two Chern-Simons terms (A.1), (A.2) are not necessarily equivalent in the presence of topological background fluxes. In fact, if we decompose $B^{\rm total} = B^{\rm bg} + B$ and $C_3^{\rm total} = C_3^{\rm bg} + C_3$, we see that the problem arises from taking either $B^{\rm bg}$ or $C_3^{\rm bg}$ to appear without a derivative in the action. In this case, the action is not necessarily gauge invariant under large gauge transformations.

As an explicit example of this problem, consider by analogy a simple U(1) gauge theory on a cubic T^3 with sides of length 1, with connection A_i and field strength $F_{ij} = \partial_i A_j - \partial_j A_i$. In this model the Chern-Simons term $\int A \wedge F$ is invariant under local gauge transformations. The topological flux F_{ij} is quantized to be $F_{ij} = 2\pi n_{ij}, n_{ij} \in \mathbb{Z}$. Let us turn on an explicit F_{12} flux by setting $A_2^{\rm bg} = 2\pi x_1$, and compute the term in the action which gives a tadpole in this background to the fluctuation $A_3 = \lambda \cos 2\pi x_1 \to F_{13} = -2\pi\lambda \sin 2\pi x_1$. This tadpole arises from the term

$$\begin{aligned} \int A_2 F_{31} &= \int_0^1 dx_1 \; (2\pi)^2 x_1 \lambda \sin 2\pi x_1 & \text{(A.3)} \\ &= -2\pi\lambda \,. & \text{(A.4)} \end{aligned}$$

We might try integrating this term by parts, in which case we get a boundary contribution

$$\int A_2^{\rm bg}(\partial_3 A_1 - \partial_1 A_3) \to -\int A_2^{\rm bg}\partial_1 A_3 \tag{A.5}$$

$$= \int F_{12}^{\rm bg} A_3 - \left(A_2^{\rm bg} A_3\right)|_0^1 \tag{A.6}$$

$$= -2\pi A_3(x_1 = 1) = -2\pi\lambda\,. \tag{A.7}$$

Thus, the integration by parts is not valid here if the boundary term is neglected. Furthermore, if we perform the global gauge transformation

$$A_i \to A_i - ig^{-1}\partial_i g\,, \tag{A.8}$$

where

$$g = e^{-2\pi i x_1 x_2}\,, \tag{A.9}$$

we have

$$A_1^{\rm bg} = -2\pi x_2\,, \tag{A.10}$$

$$A_2^{\rm bg} = 0\,. \tag{A.11}$$

The tadpole for the fluctuation $F_{13} = 2\pi\lambda \sin 2\pi x_1$ in this background explicitly vanishes! Thus, the action $\int A \wedge F$ is not invariant under large gauge transformations when the background topological flux is encoded in A which appears explicitly without derivatives in the action.

To avoid these complications, we need to find an invariant definition of the Chern-Simons term in the presence of topological fluxes. A correct definition of a D-dimensional Chern-Simons term Γ on a manifold M_D is given by finding a $(D+1)$-dimensional manifold M_{D+1} with boundary $M_D = \partial M_{D+1}$. Then

$$\int_{M_D} \Gamma = \int_{M_{D+1}} d\Gamma\,, \tag{A.12}$$

is gauge invariant under *all* gauge transformations on M_D which can be extended to gauge transformations on M_{D+1} as long as $d\Gamma$ is gauge invariant. Note that generally Γ depends on a p-form potential C, and not just on dC, so that Γ must be extended to M_{D+1} by extending C and not dC. We will use this approach to find the invariant definition of the Chern-Simons term in M-theory, which we then reduce to type IIA. A similar discussion of the Chern-Simons term of M-theory was given in [25].

To construct the Chern-Simons term of M-theory, we begin by making the simplifying assumption that we have an M_{11} which decomposes as $M_{11} = \mathbb{R} \times \hat{M}_{10}$, such that there is no topological flux of the M-theory 4-form $F_{\mu\nu\rho\sigma}$ with an index on the first dimension. We define $F^{\rm total} = dC + F^{\rm bg}$. We can then write $M_{11} = \partial M_{12}$ where $M_{11} = H_+ \times \hat{M}_{10}$ with H_+ the upper half-plane. We can then extend any C_3 from M_{11} to M_{12} by multiplying by a function of the extra coordinate which is 1 on the boundary and goes to 0 sufficiently rapidly in the interior. For example we could take e^{-r} on H_+. We extend $F^{\rm bg}$ trivially on M_{12}, which amounts to choosing a particular representative $F^{\rm bg} = dC^{\rm bg}$ and extending $C^{\rm bg}$ trivially (though note that $C^{\rm bg}$ may transform nontrivially between charts covering M_{11}). The four-form flux in 12D is then given by

$$\tilde{F} = d(e^{-r}C) + F^{\rm bg} = -dr \wedge e^{-r}C + e^{-r}F + F^{\rm bg} . \tag{A.13}$$

We can then directly integrate

$$\begin{aligned}\int_{M_{12}} \tilde{F}_4^{\rm total} \wedge \tilde{F}_4^{\rm total} \wedge \tilde{F}_4^{\rm total} &\to \int_0^\infty dr \wedge (e^{-r}C) \wedge (e^{-r}F + F^{\rm bg}) \wedge (e^{-r}F + F^{\rm bg}) \\ &= \frac{1}{3} C \wedge F \wedge F + C \wedge F \wedge F^{\rm bg} + C \wedge F^{\rm bg} \wedge F^{\rm bg} . \end{aligned} \tag{A.14}$$

The coefficient of the first term is fixed to agree with the term in the absence of background fluxes, so that using conventions of Polchinski we have

$$S_{\rm CS}^{11} = -\frac{1}{12\kappa_{11}^2} \int_{M_{11}} C_3 \wedge \left(F_4 \wedge F_4 + 3F_4 \wedge F_4^{\rm bg} + 3F_4^{\rm bg} \wedge F_4^{\rm bg} \right) . \tag{A.15}$$

This fixes the Chern-Simons term of M-theory in the presence of background fluxes as long as there is a trivial one-dimensional factor in M_{11}.

Now, we can dimensionally reduce to 10 dimensions. Following the standard dimensional reduction as in [22] but using our conventions for RR fields, we have

$$\begin{aligned} S_{\rm CS}^{\rm IIA} = -\frac{1}{2\kappa_{10}^2} \int \Big[& B_2 \wedge F_4 \wedge F_4 + 2B_2 \wedge F_4 \wedge F_4^{\rm bg} + C_3 \wedge H_3^{\rm bg} \wedge F_4 + \\ & + B_2 \wedge F_4^{\rm bg} \wedge F_4^{\rm bg} + 2C_3 \wedge H_3^{\rm bg} \wedge F_4^{\rm bg} \Big] , \end{aligned} \tag{A.16}$$

where we have integrated by parts where possible. Note that these terms reduce correctly to (A.1), (A.2) in the absence of topological fluxes. The first line of (A.16) contains all terms needed in the case of compactification of IIA on a 6-dimensional manifold, where there are no terms quadratic in the topological background flux, since this would require a nontrivial cohomology cycle of degree 7 or higher. In this case, which is the case of interest in this paper, the Chern-Simons terms are precisely those found in [14] to be compatible with the structure imposed by 4D supergravity.

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